



UNIVERSIDAD CARLOS III DE MADRID

working
papers

UC3M Working papers
Economic
13-20
December, 2013
ISSN 2340-5031

Departamento de Economía

Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 916249875

A Theory of Vintage Capital Investment and Energy Use^{*}

Antonia Díaz^a and Luis A. Puch^{b†}

^aUniversidad Carlos III de Madrid

^bUniversidad Complutense and ICAE

Abstract

In this paper we propose a theory of investment and energy use to study the response of macroeconomic aggregates to energy price shocks. In our theory this response depends on the interaction between the energy efficiency built in capital goods (which is irreversible throughout their lifetime) and the growth rate of Investment Specific Technological Change (ISTC hereafter). ISTC reduces the cost to produce investment goods and renders them more productive. Depending on which effect is stronger, higher ISTC is a complement or a substitute for energy efficiency and, thus, affects differently aggregate energy demand. Our theory provides a discipline to identify both effects since it predicts that the relative price of investment goods (not quality adjusted as well as quality adjusted) depends not only on the two ISTC shocks but also on the energy efficiency built into capital units. Thus, we can disentangle the effects of ISTC shocks from the effects of energy efficiency: a distinction that the literature on ISTC growth abstracts from. Our theory can account for the fall of energy use per unit of output observed during the 1990s, a period in which energy prices fell below trend. By increasing investment in the years of high ISTC growth, the economy was increasing the average efficiency of the economy (the capital-energy ratio), shielding the economy against the impact of the 2003-08 price shock.

Keywords: Energy use, vintage capital, energy price shocks, investment-specific technology shocks

JEL Classification: E22, E23

^{*} This paper was previously entitled "A Theory of Energy Use". We thank Raouf Boucekkine for his comments to a first version of this paper. We also thank Fabrice Collard, Andrés Erosa, Cyril Monnet, Omar Licandro, Franck Portier and Jesús Ruiz for helpful comments, and Manuel García and Alberto Sánchez for excellent research assistance. Díaz, and Puch, thank respectively the Dirección General de Investigación, projects ECO2010-20614 and ECO2010-17943, for financial support, and the EUI for its support during part of this research. Puch also thanks Fundación FocusAbengoa and FEDEA. We are grateful to participants of conferences 2012 REDg, 2012 SED Meetings, 2011 Wien Macro Workshop, as well as workshop participants at EUI.

[†] Corresponding Author: Antonia Díaz, Department of Economics, Universidad Carlos III de Madrid, 28093 Madrid, Spain; E-mail: andiaz@eco.uc3m.es

1 Introduction

The two oil crises in the 1970s and a growing environmental awareness of societies and governments have prompted many macroeconomists to study the determinants of aggregate energy use and, therefore, the response of macroeconomic aggregates to changes in energy prices.¹ Early studies estimated a very low short run price elasticity of energy demand, which accounted for the large impact of the oil price shocks in the 1970s (see Berndt and Wood 1975, Griffin and Gregory 1976, and Pindyck 1979). The same authors also report that this elasticity is higher in the long run, as the economies reduce their energy consumption slowly. More recently, Newell et al. (1999), and Popp (2002) find that energy prices have a strong positive effect on energy-efficient technical innovations, suggesting that the difference between the short and long run price elasticities of energy demand is due to the ability of the economies to move to more energy-efficient technologies.² Lately, though, it has been pointed out that innovation in energy efficiency is affected, even delayed, by the level of Investment Specific Technical Change (ISTC hereafter); see, for instance Steinbuks and Neuhoff (2010), or Knittel (2011).³ This evidence suggests three things: First, that energy demand is very much related to the efficiency of the technologies chosen; second, that the level of energy efficiency responds to changes in energy prices and, third, that the existence of Investment Specific Technical Change (ISTC hereafter) affects the magnitude of that response. Following this line of thought, in this paper we propose a theory of vintage capital investment to investigate the importance of ISTC growth in shaping the response of energy demand (and use) and macroeconomic aggregates to energy price shocks.

In our model economy capital is irreversible and heterogeneous in two dimensions: its vintage and its energy-efficiency. As in Atkeson and Kehoe (1999), the energy efficiency of a unit of capital is given by the energy required for this capital to yield services. Higher efficiency comes at the cost of less capital services and, therefore, less output. Unlike Atkeson and Kehoe (1999), we assume that there is ISTC growth as in Solow (1960). Hence, aggregate energy use per unit of output may fall because either new capital is more energy efficient or is more productive per unit of energy

¹See Hamilton (2008) or Kilian (2008) for a survey of the significant literature.

²Newell et al. (1999) estimate that during the period 1958-1993 about half of the total change in energy efficiency of the models of room air conditioners, central air conditioners, and gas water heaters was prompted by the high price shocks experienced during the 1970s and early 1980s.

³Knittel (2011) estimates that if car quality would have not improved from 1980 to 2006, energy efficiency of both passenger cars and light trucks could have increased by nearly 60 percent from 1980 to 2006, instead of the observed meager 15 percent.

consumed, or both things at the same time. To focus our study in the determinants of energy use and efficiency, we assume that all energy is imported and that the energy price is exogenously given. Moreover, there is no rationing: the amount of energy demanded at any given price is always satisfied. This is a closed economy in any other respect.

We start by studying the properties of the efficient allocation. We show that we can decentralize it by opening a secondary market for capital goods. In our economy the market price of one unit of capital depends not only on its vintage but also on its energy efficiency. Standard aggregation (i.e., representing aggregate value added as a function of aggregate capital and labor) is not possible in this economy, in general. This is so because the complementarity of capital and energy at the micro level implies that the energy share in aggregate gross output varies with the distribution of capital across vintages and efficiency levels. We can, however, establish conditions under which there is a representation for aggregate gross output as a function of aggregate labor, capital, and energy. This representation is obtained by aggregating capital using prices in units of gross output that are, essentially, cost production prices. This procedure is consistent with the method used by the Bureau of Economic Analysis (and other statistical agencies in different countries) to construct the aggregate stock of capital of the US economy. The aforementioned cost production prices depend on the vintage as well as the energy efficiency of capital. We also find aggregate capital adjusted by quality and its corresponding price. Consistently with the estimates by Gordon (1990) and Cummins and Violante (2002), this price falls with the level of ISTC, but it rises with the level of energy efficiency. That is, the relative price of capital goods falls less rapidly in periods in which the economy is investing more in energy efficiency, as it happened in the 1970s. Thus, we provide a theoretical foundation to the view expressed by Gordon (1996), who argued that changes in energy efficiency is one of the key determinants of the dynamics of the relative price of investment goods. Finally, we show that our theory encompasses previous theories of investment and energy use: Atkeson and Kehoe (1999) and Díaz et al. (2004) are special cases of our economy without ISTC growth.

The aggregate representation of our economy shows very starkly that the existence of ISTC has two effects of opposite sign on aggregate energy use. On the one hand, new investment is needed to rip the benefits of higher ISTC, which rises the absolute level of energy use. On the other hand, higher ISTC, *ceteris paribus*, rises capital productivity and output, which lowers the level of energy use per unit of output. Depending on which effect is higher, ISTC can be thought of as an

energy-consuming or an energy-saving technical device. In the first case ISTC and energy efficiency are complements, in the second case they are substitutes. This complex interaction of ISTC and energy efficiency—which also depends on the behavior of energy prices—cannot be ascertained theoretically. This is why we resort to study the dynamics of aggregate energy use and energy efficiency in our model economy quantitatively.

We calibrate our model economy to match selected statistics of the US economy for the period 1960-2008. We assume that there are energy price shocks as well as two ISTC shocks. The two ISTC shocks affect the cost of producing capital goods and the level of technical progress embodied in new capital units. We disentangle these two shocks by exploiting the fact that the BEA adjusts for quality the price of some (but not all) investment goods. We prefer to abstract from neutral technological progress to isolate the effects of ISTC shocks on energy efficiency and use. We estimate the three processes so that the relative price of energy and the relative prices of investment—both the non quality-adjusted as well as the quality-adjusted price—in the model have the statistical properties of their counterparts in the data. The identification of the two ISTC shocks becomes a central issue in order to assess the quantitative properties of our model economy. It turns out that our theory gives us discipline to implement a novel way to identify both shocks, provided that only one of them is permanent. We can identify them because, according to our theory, the relative price of investment goods (not quality adjusted as well as quality adjusted) depends not only on the two ISTC shocks but also on the energy efficiency built into capital units. Hence, we can disentangle the effects of ISTC shocks from the effects of energy efficiency. It is interesting to note that the innovations in ISTC shocks are larger and highly correlated right after the oil price shocks during the early 1980s and during the late 1990s, right before the 2003-08 oil price shock. In order to assess the importance of ISTC we compare the evolution of energy use and aggregate energy efficiency in our simulated economy and a version of Atkeson and Kehoe (1999) (which is essentially our economy without ISTC) to the observed evolution of energy use and efficiency in the US economy for the aforementioned periods.

Our simulations show that investment in energy efficiency depends on the persistence of energy price shocks and the nature and persistence of ISTC shocks. If energy prices are high and ISTC in the quality of new investment are low, agents prefer to invest in energy efficient capital goods. This is the regime observed in the 1970s and 1980s, where energy use per unit of output and the average efficiency in the economy are negatively and positively correlated with energy prices, respectively.

If energy prices are low and ISTC shocks to the quality of new investment are high and transitory, agents invest in less efficient capital goods. In this case the correlation of energy use per unit of output and average energy efficiency with energy prices is reversed. Thus, our model economy can explain the regime change experienced during the 1990s, period of accelerated ISTC growth. A model economy without ISTC growth, as Atkeson and Kehoe (1999), cannot explain this change of regime, since in that economy the level of energy efficiency is solely governed by energy prices. Finally, we show that the short run price elasticity of energy use per unit of value added is equal to the value of the energy share in value added. Thus, we can quantify the regime change experienced. For instance, at the peak years 1981 and 2008 the real relative price of energy was about the same, a bit higher in 2008. The energy share, though, was 8.89 percent and 6.66 percent of value added, respectively. Thus, according to our theory, energy use has been less responsive to changes in energy prices during the last oil price shock because energy efficiency is much higher than in previous decades. Thus, our theory offers an explanation about the apparent softened response of GDP to the 2003-08 oil price shock. See for instance, Kilian (2008), or Blanchard and Galí (2007).

We view ours as a theory of investment where we make explicit the determinants of energy use and energy-saving technical change. Thus, ours has a flavor of a theory of directed technical change, as Acemoglu (2002), and Acemoglu et al. (2012), although the former studies the determinants of skill biased technical change whereas the latter focus on the substitution from environmental damaging to friendly technologies. A related work to ours is that by Hassler et al. (2012), who build on the tradition of Pindyck and Rotemberg (1983) and use an aggregate production function to calibrate a measure of energy-saving technical change consistent with the US experience, although they ignore the existence of Investment Specific Technical Change.

The organization of the paper is as follows. Section 2 describes our model economy and defines a quasi-social planner problem whose solution is the efficient allocation of our economy. In Section 3 we discuss the role of investment specific technological change. In Section 4 we describe how we calibrate our economy and our procedure to identify investment specific technological shocks. Section 5 presents our main results about the time series properties of our model economy in connection with the evidence. Section 6 concludes.

2 The benchmark model economy

Here we present our benchmark model economy where investment specific technological change plays a central role.

2.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\ell_t)), \quad \beta \in (0, 1) \quad \xi > 0, \quad (2.1)$$

where c_t is consumption and ℓ_t is leisure t . Each household is endowed with \bar{h} units of time and, therefore, works $\bar{h} - \ell_t$ hours every period.

2.2 Technology and the physical environment

Production of the unique final good is carried out at a continuum of autonomous plants. A plant is created by installing one unit of capital. Plants are indexed by the vintage of the unit of capital installed, denoted by z , and the energy efficiency with which that unit of capital must be operated, denoted by v . Output is produced combining labor and the services of the unit of capital installed according to the technology

$$y_t(z, v) = A_t \kappa_t(z, v)^\alpha h_t(z, v)^{1-\alpha}, \quad z \leq t, \quad v \in \mathbb{R}_{++}, \quad (2.2)$$

with $\alpha \in (0, 1)$. A_t is the neutral technological change factor; whereas $\kappa_t(z, v)$ and $h_t(z, v)$ are, respectively, the amount of services provided by the unit of capital installed and the amount of labor services employed in the plant.

As we have already mentioned, capital is heterogeneous in two dimensions: Its vintage, z , and its energy efficiency, v . The vintage is given by the date at which the unit of capital was produced. Thus, $z \leq t$. The efficiency type v takes values in \mathbb{R}_{++} . In order to yield services the unit of capital needs to be combined with energy. The type v determines the amount of energy, $e_t(z, v)$, needed to

produce the amount $\kappa_t(z, v)$ of capital services:

$$\kappa_t(z, v) = \Lambda_z \Gamma_z v^{1-\mu} \min \{e_t(z, v), \Gamma_z^{-1} v^\mu\}, \quad z \leq t, \quad v \in \mathbb{R}_{++}, \quad (2.3)$$

where $\mu > 1$. Λ_z and Γ_z refer to embodied investment specific technical change. As shown in Figure 1, energy efficiency type v and embodied investment specific technological change are meant to refer to different factors. We may think of type v as engine power of a car. When the car is used at optimal speed the energy consumed is $\Gamma_z^{-1} v^\mu$. In this case, the services yielded by the unit of capital are $\kappa_t(z, v) = \Lambda_z v$. Since $\mu > 1$, energy consumption at optimal speed is more than proportional to engine power. When the amount of energy consumed is less than the amount required to drive at optimal speed, the amount of services decreases with the efficiency type v , $\kappa_t(z, v) = \Lambda_z \Gamma_z v^{1-\mu} e_t(z, v)$, although it increases with energy consumption. This assumption implies that a SUV car may yield less services when driven in downtown, at suboptimal speed, than a smaller car equipped with less engine power. Λ_z refers to technological improvements that increase capital services which are not directed to save energy, whereas Γ_z refers to innovations that save energy directly. For instance, any improvement in the ergonomics of car seats, so that driving is less tiring, would be an increase in Λ_z , whereas improvements in the aerodynamics of the car would rise its energy efficiency, and should be considered as increases in Γ_z .⁴ Thus, embodied technological progress has a dual nature in our theory: it augments productivity and lowers energy consumption given the engine power, v . Either factor, Λ_z , and Γ_z , may vary stochastically over time in a manner specified in Section 4. For simplicity, we will call a pair (z, v) a *technology class*, and $\kappa_t(z, v)$ will be the amount services yielded by a unit of capital of class (z, v) at period t . Likewise, $k_t(z, v)$ is the amount of capital of class (z, v) at time t .

Additionally, there is a technology that allows agents to transform final good of period t into Θ_t units of capital of vintage $t + 1$. Thus, as it is standard in the literature, we take the view that investment specific technological change not only brings higher quality but also the production of capital goods becomes increasingly efficient with the passage of time (see, for instance, Greenwood et al. 1997). Specifically, we will denote as $x_t(v)$ the amount of final good invested in capital of vintage $t + 1$ that will be operated with efficiency type $v \in \mathbb{R}_{++}$ at period $t + 1$. The factor Θ_t may vary stochastically over time. Thus, notice that changes in Λ_{t+1} , Γ_{t+1} or Θ_t shift the technological frontier whereas changes in v are movements along the technological frontier.

⁴See Knittel (2011) for particular examples of both types of embodied technical innovations.

The number of plants of class (z, v) is equal to the amount of capital of that class, $k_t(z, v)$. Notice that while the vintage is given exogenously by the time at which the unit of capital was produced, the measure of classes respond to economic conditions. Capital is irreversible; that is, capital of vintage z_1 cannot be converted in capital of vintage z_2 . Likewise, capital is irreversible across efficiency types. Capital of vintage z of efficiency type v_1 cannot be transformed into type v_2 . This amounts to saying that plants of class (z, v) cannot be converted into plants of a different class. The plant, though, can be left idle by not allocating either energy or labor. Finally, once production has taken place, the plant faces a positive probability of death, $\varpi \in [0, 1]$, which is i.i.d. across plants. This death implies the destruction of the unit of capital. This death probability plays the role of physical depreciation of capital. Therefore,

$$k_t(z, v) = (1 - \varpi)^{t-z} k_z(z, v), \quad z \leq t, \quad v \in \mathbb{R}_{++}. \quad (2.4)$$

2.3 Energy consumption and energy prices

We will assume that energy is entirely bought in an international market at an exogenously given price q_t . Therefore, from the point of view of the economic agents, the energy price follows a stochastic process. We assume that there is no international borrowing and lending. In absence of an international credit market we can think of the price of energy as given by nature. This implies that, under market completeness, the Second Welfare Theorem applies and, therefore, we can restrict our attention to efficient allocations.

2.4 Aggregate value added

The amount of aggregate labor used in the production of the final good satisfies

$$0 \leq \ell_t \leq \bar{h} - \sum_{z=-\infty}^t \int_0^{\infty} k_t(z, v) h_t(z, v) dv, \quad 0 \leq h_t(z, v) \leq \bar{h}, \quad z \leq t, \quad v \in \mathbb{R}_{++}, \quad (2.5)$$

Aggregate value added, va_t , is aggregate production of the final good net of energy expenditures. The feasibility constraint is

$$c_t + \int_0^\infty x_t(v) dv \leq va_t \equiv \sum_{z=-\infty}^t \int_0^\infty k_t(z, v) [y_t(z, v) - \varrho_t e_t(z, v)] dv, \quad (2.6)$$

where c_t denotes consumption, and $\int_0^\infty x_t(v) dv$ is aggregate investment. Thus,

$$0 \leq k_{t+1}(t+1, v) \leq \Theta_t x_t(v). \quad (2.7)$$

2.5 Properties of the efficient allocation

The efficient allocation is found by maximizing (2.1) subject to (2.3)–(2.7). The planning problem is fully specified in Appendix B. In Appendix C we show the decentralized version of our economy. There we show that we can price capital of all classes by opening a secondary market for plants. In what follows we are going to characterize the efficient allocation and provide conditions that guarantee that the economy can be aggregated.

Lemma 1. *Marginal productivity of labor is the same across all plants operated:*

$$(1 - \alpha) A_t \kappa_t(z, v)^\alpha h_t(z, v)^{-\alpha} = \frac{u_\ell(c_t, \ell_t)}{u_c(c_t, \ell_t)}. \quad (2.8)$$

This is a direct consequence of labor being freely mobile across plants. In the decentralized version of our economy (shown in Appendix C) the price of one unit of labor services is $w_t \equiv \frac{u_\ell(c_t, \ell_t)}{u_c(c_t, \ell_t)}$. Thus, Lemma 1 implies that the labor share in gross output is the same in all plants. Now we turn to investigate which classes of capital are allocated energy.

Proposition 1. *If at time t a plant of class (z, v) , $z \leq t$, $v > 0$, is operated, $e_t(z, v) > 0$, then it is operated at the requirement level, $e_t(z, v) = \Gamma_z^{-1} v^\mu$ and $\kappa_t(z, v) = \Lambda_z v$.*

Proof. Consider the Lagrangian

$$\max_{e_t(z, v)} A_t (\Lambda_z \Gamma_z v^{1-\mu} e_t(z, v))^\alpha h_t(z, v)^{1-\alpha} - \varrho_t e_t(z, v) + \Psi_t^{e_1}(z, v) (\Gamma_z^{-1} v^\mu - e_t(z, v)) + \Psi_t^{e_0}(z, v) e_t(z, v).$$

(2.9)

For any plant for which $h_t(z, v) > 0$, if $0 < e_t(z, v) < \Gamma_z^{-1} v^\mu$, then $\Psi_t^{e_0}(z, v) = 0$, and it must be the case that

$$\alpha A_t (\Lambda_z \Gamma_z v^{1-\mu})^\alpha e_t(z, v)^{\alpha-1} h_t(z, v)^{1-\alpha} - \varrho_t = \Psi_t^{e_1}(z, v). \quad (2.10)$$

Using (2.3) and (2.8), expression (2.10) becomes

$$\alpha A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}} \Lambda_z \Gamma_z v^{1-\mu} - \varrho_t = \Psi_t^{e_1}(z, v), \quad (2.11)$$

where $w_t \equiv \frac{u_\ell(c_t, \ell_t)}{u_c(c_t, \ell_t)}$. Notice that expression (2.11) does not depend on $e_t(z, v)$. Thus if $\Psi_t^{e_1}(z, v)$ is positive, then $e_t(z, v) = \Gamma_z^{-1} v^\mu$, and $\kappa_t(z, v) = \Lambda_z v$. \square

Thus, it is never efficient to operate a plant using less energy than the requirement level. Proposition 1 and expressions (2.2) and (2.8) imply that production at the plant level, relative to production of the latest vintage, only depends on its class:

Corollary 1. *The amount of output produced and labor used, relative to the aggregate, in a plant only depend on the class of the plant; thus,*

$$\frac{h_{t+i}(z, v_z)}{h_{t+i}(t, v_t)} = \frac{y_{t+i}(z, v_z)}{y_{t+i}(t, v_t)} = \frac{\Lambda_z v_z}{\Lambda_t v_t}, \text{ for all } i \geq 0. \quad (2.12)$$

This Corollary describes how labor is allocated across plants. Labor is withdrawn gradually from older and less efficient plants and placed in newer and more efficient plants. Therefore, output produced by older and less efficient classes is reduced gradually. Labor (and energy) allocated to plants becomes zero whenever production is too low compared to the required energy expenditure. The cut-off point is implied by the non-negativity of the first order condition shown in equation (2.11) in Proposition 1.

Corollary 2. *For a plant of class (z, v) to be utilized in equilibrium it must be the case that*

$$\alpha A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}} \Lambda_z v - \varrho_t \Gamma_z^{-1} v^\mu \geq 0, \quad (2.13)$$

where $w_t \equiv \frac{u_\ell(c_t, \ell_t)}{u_c(c_t, \ell_t)}$. For each vintage z there exists a type \underline{v}_{zt} for which (2.13) holds with strict equality. The type \underline{v}_{zt} decreases with ϱ_t and increases with z .

In the decentralized version of this economy, expression (2.13)—which is the same that the first order condition shown in (2.11)—is the profit accrued in a plant of class (z, v) . Thus, expression (2.13) says that for a plant to be utilized, profit has to be non-negative. This profit is equal to the return to capital net of energy expenditure, and it increases with the level of embodied technical change, Λ_z and Γ_z , and decreases with v . Profit accrued is zero at the marginal type \underline{v}_{zt} .

The difference in energy expenditure across classes of capital has implications at the aggregate level. The share of energy in aggregate value added (and, therefore, those of capital and labor) depends on the entire distribution of capital across classes. This property will have consequences for the aggregation of our economy. Let us turn now to analyze the investment decision and the characteristics of new plants.

Proposition 2. *If all installed capital is always utilized, $e_t(z, v) > 0$, for all t , z , and v , then, all units of new capital are operated with the same level of energy efficiency, $v_{t+1} > 0$.*

Proof. The first order condition with respect to investment allocated to type v , $x_t(v)$, is

$$\Theta_t^{-1} \varphi_t - \Psi_t^k(v) = E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \varphi_{t+i} [\alpha y_{t+i}(t+1, v) - \varrho_{t+i} e_{t+i}(t+1, v)], \quad (2.14)$$

where φ_t is marginal utility of consumption at time t . Investment in type v is positive only if $\Psi_t^k(v) = 0$. This multiplier is non negative so zero is its minimum value. Thus, we have to show that $\Psi_t^k(v)$ has a unique minimum. Equivalently, present value of capital income, shown in the right hand side of (2.14), has a unique maximum. Using (2.8) and the result of Proposition 1 and plugging them in the expression of output at the plant level, (2.2), we can write (2.14) as

$$\Theta_t^{-1} \varphi_t - \Psi_t^k(v) = E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \varphi_{t+i} \left[\alpha A_{t+i}^{\frac{1}{\alpha}} \left(\frac{1 - \alpha}{w_{t+i}} \right)^{\frac{1-\alpha}{\alpha}} \Lambda_{t+1} v - \varrho_{t+i} \Gamma_{t+1}^{-1} v^\mu \right], \quad (2.15)$$

where $w_{t+i} \equiv \frac{u_\ell(c_{t+i}, \ell_{t+i})}{u_c(c_{t+i}, \ell_{t+i})}$. The right hand side of (2.15) has a unique maximum in v , that we call v_{t+1} , and it satisfies

$$\frac{\alpha}{\mu} E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \varphi_{t+i} y_{t+i}(t+1, v_{t+1}) = E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \varphi_{t+i} \varrho_{t+i} \Gamma_{t+1}^{-1} v_{t+1}^\mu, \quad (2.16)$$

that is, the present value of all future energy expenditures in a plant that uses a unit of capital of vintage $t + 1$ is the fraction α/μ of the present value of future gross output produced by that plant. Hence the price of one unit of new capital, Θ_t^{-1} , must be equal to the fraction $\alpha(\mu - 1)/\mu$ of the present value of future gross output. \square

This Proposition shows that if all plants are always utilized, it is welfare improving to place all new investment in only one efficiency type, the one that maximizes the present value of future output produced in the plant net of energy expenditures and imputed labor income. Notice that manipulating (2.16) we obtain that the efficiency v_{t+1} satisfies

$$v_{t+1}^{\mu-1} = \Lambda_{t+1} \Gamma_{t+1} \frac{E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \varphi_{t+i} \alpha A_{t+i}^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_{t+i}} \right)^{\frac{1-\alpha}{\alpha}}}{\mu E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \varphi_{t+i} \varrho_{t+i}}. \quad (2.17)$$

Thus, at any time t investment will be allocated to a unique technology class $(t + 1, v_{t+1})$ and in period $t + 1$ all capital of vintage $t + 1$ will be operated with the same energy efficiency.

2.6 Aggregation

We would like to represent aggregate value added as a function of aggregate capital. In order to aggregate capital we need to define appropriate relative prices for each class. It is easy to do so for the latest vintage produced, since its shadow price must be equal to the inverse of the productivity in the production of capital goods, Θ_t^{-1} , but we do not have an equivalent measure for the previous vintages. Therefore, we define a price for any class of capital in the following way:

Definition 1. *Let the cost of one unit of capital of class (z, v) , $z \leq t + 1$, $v_z \in \mathbb{R}_{++}$, in units of gross output at time t be defined as*

$$q_t(z, v_z) \equiv \Theta_t^{-1} \frac{\Lambda_z}{\Lambda_{t+1}} \frac{v_z}{v_{t+1}}, \quad z \leq t + 1, \quad v \in \mathbb{R}_{++}. \quad (2.18)$$

Notice that capital goods of the same vintage may have different prices, depending on their energy efficiency. This is consistent with Gordon (1990, 1996), who argued that not all changes in the

relative price of capital goods are due to investment specific technical change but also to changes in energy efficiency (see Gordon 1996, p. 262).

We need to emphasize that the price defined above is not the market price of a unit of capital of class (z, v) at time t , since it does not measure the present value of the expected return to one unit of capital net of energy expenditures. The price shown in (2.18) is the cost of capital of class (z, v_z) in units of gross output at period t . Appendix C shows the decentralized version of this economy and that the market price of one unit of capital of class (z, v) at time $t \geq z$ is

$$p_t(z, v_z) = \frac{\Lambda_z v_z}{\Lambda_{t+1} v_{t+1}} \Theta_t^{-1} + \left(\frac{\Lambda_z v_z}{\Lambda_{t+1} v_{t+1}} - \frac{\Gamma_z^{-1} v_z^\mu}{\Gamma_{t+1}^{-1} v_{t+1}^\mu} \right) \frac{\Theta_t^{-1}}{\mu - 1}. \quad (2.19)$$

This price takes into account the associated cost of energy; that is, $p_t(z, v_z)$ is the price of capital in units of value added. Notice that this price is higher than the cost price $q_t(z, v_z)$ if

$$\frac{\Lambda_z v_z}{\Gamma_z^{-1} v_z^\mu} > \frac{\Lambda_{t+1} v_{t+1}}{\Gamma_{t+1}^{-1} v_{t+1}^\mu}. \quad (2.20)$$

That is, if the ratio of capital services to energy for class (z, v_z) is higher than the corresponding ratio for the new capital of class (z, v_z) . Unless all capital is operated with the same energy efficiency the market price of capital and its cost in units of gross output are different.

It could be argued that we should find aggregate capital using market prices. We would do so if we could represent aggregate value added solely as a function of labor and capital, which we cannot do because of the complementarity of capital and energy at the plant level. This complementarity implies that factor shares in aggregate value added depend on the distribution of capital across vintages. As a consequence, we cannot write aggregate value added as a function of aggregate primary inputs. We can, however, find aggregate gross output since the share of labor in gross output is constant regardless of the capital vintage and energy efficiency.⁵

⁵See Sato (1976) for a more detailed discussion on the issue of aggregation.

2.6.1 Aggregate capital and its relative price

Let us define as k_t the aggregate volume of capital, in per capita terms, in units of the latest class. Thus,

$$k_t = \sum_{z=-\infty}^t \frac{q_t(z, v_z)}{q_t(t, v_t)} k_t(z, v_z). \quad (2.21)$$

Taking into account that investment takes place in only one class of capital, we can express capital as

$$k_t = \sum_{z=-\infty}^t \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} (1 - \varpi)^{t-z} \Theta_{z-1} x_{z-1}. \quad (2.22)$$

The product $\frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} (1 - \varpi)^{t-z}$ can be interpreted as the remaining value of the stock of class (z, v_z) (per unit of capital) once physical depreciation and obsolescence are taken into account. In other words, the average depreciation rate of capital class (z, v_z) at time t is

$$\delta_t(z, v_z) = 1 - (1 - \varpi) \left(\frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} \right)^{\frac{1}{t-z}}. \quad (2.23)$$

The average relative price (cost) of capital in units of consumption good is, by definition of k_t , equal to the relative price of vintage t and type v_t .

$$q_t = \sum_{z=-\infty}^t q_t(z, v_z) \frac{k_t(z, v_z)}{k_t} = \frac{\Lambda_t}{\Lambda_{t+1}} \frac{v_t}{v_{t+1}} \Theta_t^{-1}. \quad (2.24)$$

We can also find the aggregate amount of capital services or, in other words, the amount of quality-adjusted capital:

$$\kappa_t = \Lambda_t v_t \sum_{z=-\infty}^t q_t(z, v_z) k_t(z, v_z) = \Lambda_t v_t k_t. \quad (2.25)$$

Now we can define the cost of quality-adjusted capital; that is, the cost of capital services,

Definition 2. Let the cost of one unit of capital services of class (z, v_z) , $z \leq t + 1$, $v_z \in \mathbb{R}_{++}$, in units of gross output at time t be defined as

$$q_t^\kappa \equiv \frac{q_t(z, v)}{\lambda_z v_z}, \quad z \leq t + 1, \quad v \in \mathbb{R}_{++}. \quad (2.26)$$

This price is equal to $\Theta_t^{-1} \Lambda_{t+1}^{-1} v_{t+1}^{-1}$ for all classes of capital.

Consistently with Gordon (1990) and Cummins and Violante (2002), this price falls when ISTC rises, as measured by $\Theta_t \Lambda_{t+1}$, but it rises when energy efficiency rises; that is, when v_{t+1} falls. Thus, when agents invest in improving energy efficiency of new capital, the relative price of new capital rises, as it was suggested by Gordon (1996). Thus, our theory says that we cannot attribute all changes in the relative price of investment to ISTC but also to changes in energy efficiency.

This method to define the aggregate net stock of capital is consistent with the procedure used by the Bureau of Economic Analysis (and other statistical agencies in different countries) to measure the real value of the net stock of capital. The only difference is that the BEA defines the stock of capital in units of output; in our notation:

$$\tilde{k}_t = \sum_{z=-\infty}^t (1 - \varpi)^{t-z} \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} \frac{\Theta_{z-1}}{\Theta_{t-1}} x_{z-1} = \sum_{z=-\infty}^t (1 - \varpi)^{t-z} \frac{q_{t-1}^\kappa}{q_{z-1}^\kappa} x_{z-1}. \quad (2.27)$$

Notice that \tilde{k}_t is equal to our stock of quality-adjusted capital valued at prices at the beginning of the period, $\tilde{k}_t = q_{t-1}^\kappa \kappa_t$, which is equal to $\Theta_{t-1}^{-1} k_t$. The BEA depreciates investment realized at time $z-1$ by applying economic depreciation rates which include physical decay as well as obsolescence.⁶ In terms of our theory, the obsolescence component of the depreciation rate must be given by the change in the quality-adjusted relative price of investment, $q_{t-1}^\kappa / q_{z-1}^\kappa$. As a matter of fact, this is the procedure followed by Cummins and Violante (2002) to calibrate the physical depreciation rate of capital. We add to their analysis by providing a theory of how the relative price of investment depends on energy efficiency and, therefore, energy prices.

2.6.2 Output, hours worked and energy use

Production of all plants of class (z, v) is the expected output of a plant of vintage z which, using (2.12) can be written as

$$y_t = \sum_{z=-\infty}^t \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} k_t(z, v_z) y_t(t, v_t). \quad (2.28)$$

⁶See Bureau of Economic Analysis (2003) for a detailed description of the perpetual inventory method used to construct the aggregate stock of capital measure.

Likewise, using (2.8) and (2.18) we can write aggregate labor as

$$h_t = \sum_{z=-\infty}^t \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} k_t(z, v_z) h_t(t, v_t) = k_t h_t(t, v_t). \quad (2.29)$$

Aggregate gross output is

$$y_t = A_t \Lambda_t^\alpha (v_t k_t)^\alpha h_t^{1-\alpha}. \quad (2.30)$$

Likewise, aggregate energy consumption is

$$e_t = \sum_{z=-\infty}^t \Gamma_z^{-1} v_z^\mu k_t(z, v_z). \quad (2.31)$$

2.6.3 The aggregated economy

The law of motion of capital is

$$k_{t+1} = \Theta_t x_t + (1 - \varpi) \frac{\Lambda_t}{\Lambda_{t+1}} \frac{v_t}{v_{t+1}} k_t. \quad (2.32)$$

Since all plants are utilized in equilibrium, the law of motion of energy consumption is

$$e_{t+1} = \Gamma_{t+1}^{-1} v_{t+1}^\mu \Theta_t x_t + (1 - \varpi) e_t. \quad (2.33)$$

The quasi-social planner's problem as

$$\begin{aligned} \max_{\substack{c_t, x_t, \\ v_{t+1}}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\bar{h} - h_t)) \\ \text{s. t.} \quad & c_t + x_t \leq A_t (\Lambda_t v_t k_t)^\alpha h_t^{1-\alpha} - \varrho_t e_t, \\ & e_{t+1} \geq \Gamma_{t+1}^{-1} v_{t+1}^\mu \Theta_t x_t + (1 - \varpi) e_t, \\ & 0 \leq k_{t+1} \leq \Theta_t x_t + (1 - \varpi) \frac{\Lambda_t}{\Lambda_{t+1}} \frac{v_t}{v_{t+1}} k_t, \\ & k_0 \text{ given, } v_{t+1} \geq 0, x_t \geq 0. \end{aligned} \quad (2.34)$$

Notice that at time t , the amount of energy consumed is a state variable. This is a direct consequence of all capital installed being used in equilibrium. It will be useful to write the quasi-

social planner's problem in terms of quality-adjusted capital using (2.25):

$$\begin{aligned}
& \max_{\substack{c_t, x_t, \\ v_{t+1}}} E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\bar{h} - h_t)) \\
& \text{s. t.} \quad c_t + x_t \leq A_t \kappa_t^\alpha h_t^{1-\alpha} - \varrho_t e_t, \\
& \quad e_{t+1} \geq \Gamma_{t+1}^{-1} v_{t+1}^\mu \Theta_t x_t + (1 - \varpi) e_t, \\
& \quad 0 \leq \kappa_{t+1} \leq v_{t+1} \Lambda_{t+1} \Theta_t x_t + (1 - \varpi) \kappa_t, \\
& \quad \kappa_0 \text{ given, } v_{t+1} \geq 0, x_t \geq 0.
\end{aligned} \tag{2.35}$$

Hence, by solving this planner's problem we can find the aggregates of our decentralized economy. Notice that the evolution of the aggregate stock of capital is shown in (2.32). Finally, we need to add the condition that ensures that this aggregation works: all installed capital is used in equilibrium. Using (2.13) with strict inequality, we can write the following assumption:

Assumption 1. *The energy price is never too large,*

$$\alpha A_t \kappa_t^{\alpha-1} h_t^{1-\alpha} \Lambda_z v_z > \varrho_t \Gamma_z^{-1} v_z^\mu, \text{ for all } z \leq t+1. \tag{2.36}$$

Expression (2.36) is equivalent to (2.13) with strict inequality. Multiplying both sides of the inequality by $k_t(z, v_z)$ and adding across vintages the inequality becomes

$$\alpha A_t \kappa_t^\alpha h_t^{1-\alpha} > \varrho_t e_t. \tag{2.37}$$

That is, the share of energy in value added cannot exceed gross capital income.

3 The role of investment specific technological change

In this section we want to highlight the effect of investment specific technical change on the choice of energy efficiency. To this purpose, we want to compare our theory with that of Atkeson and Kehoe (1999) and our previous work Díaz et al. (2004).

3.1 The cost of saving energy in a putty-clay model economy

Let us turn now to the economy proposed by Atkeson and Kehoe (1999). Their economy is essentially ours in absence of ISTC but, still, capital is irreversible. We show in Appendix F the mapping of their economy to our economy in detail. The evolution of the microeconomic aggregates is given by the following equations:

$$c_t + x_t = A_t \kappa_t^\alpha h_t^{1-\alpha} - \varrho_t e_t, \quad (3.1)$$

$$e_{t+1} = v_{t+1}^\mu x_t + (1 - \varpi) e_t, \quad (3.2)$$

$$\kappa_{t+1} = v_{t+1} x_t + (1 - \varpi) \kappa_t. \quad (3.3)$$

In this economy capital and energy are complements in the short run. To go straight to the point, assume that agents do not value leisure so that $h_t = \bar{h}$ every period. Thus, right after an unexpected positive energy price shock at a given period t , the economy cannot reduce its energy consumption (and, therefore, its energy bill) so that the immediate effect is a fall in aggregate value added, $y_t - \varrho_t e_t$. If the energy price has persistence, the expected price will rise, too. This reduces the future return to current investment. The only way that this economy has to reduce its energy consumption in future periods is by investing less (lower x_t) or by making new capital more efficient; i.e., by reducing v_{t+1} and increasing the capital-energy ratio. In either case the impact of an energy price rise is propagated into the future as less capital services and, therefore, less aggregate value added. By making capital more energy-efficient the economy reduces its energy bill. This implies that energy use (i.e., energy consumption per unit of value added) falls. Thus, energy use and the capital to energy ratio will track the evolution of the energy price, with a delay which will be longer the lower the depreciation rate ϖ . Moreover, the relative price of quality-adjusted capital, which is just equal to v_{t+1}^{-1} , will increase, reflecting the rise in the energy price. Let us turn now to examine our economy with investment specific technological change.

3.2 Investment specific technological change and energy efficiency

Let us turn now to our economy with ISTC. The expression of aggregate gross output and aggregate value added is the same as in the Atkeson and Kehoe's economy. The evolution of capital services

and energy consumption is given by the expressions:

$$e_{t+1} = \Gamma_{t+1}^{-1} v_{t+1}^{\mu} \Theta_t x_t + (1 - \varpi) e_t, \quad (3.4)$$

$$\kappa_{t+1} = v_{t+1} \Lambda_{t+1} \Theta_t x_t + (1 - \varpi) \kappa_t. \quad (3.5)$$

The first thing that stands out in the laws of motion shown above is that ISTC have different effects on the evolution of energy consumption and quality-adjusted capital. An ISTC change have a dual effect: it allows to produce more machines and it allows to produce better machines at the same cost in units of output. In our economy the first effect is captured by the Θ_t shock and the second one is captured by Λ_{t+1} and Γ_{t+1} . Higher Θ_t or lower Γ_{t+1} entail higher energy consumption, whereas higher Λ_{t+1} entails higher capital services and lower energy use (i.e., per unit of value added). Thus, ISTC and energy efficiency are complements or substitutes depending on the nature of the ISTC shocks. If embodied technical shocks, Λ_{t+1} and Γ_{t+1} , are low relative to shocks in the cost of producing capital, Θ_t , an energy price rise will prompt investing in energy efficiency, more so that in an economy in which all technical progress is neutral. On the other hand, if the price falls in a period in which there is an acceleration in embodied technical change, the economy will not invest in energy efficiency, less so that in an economy in which all technical progress is neutral. We will turn to discuss all these interactions in more detail in section 5, where we illustrate quantitatively the ability of our model economy to account for the observed dynamics of energy use, the capital-energy ratio and other macroeconomic aggregates.

3.3 Capital adjustment costs as a short-cut for capital irreversibility

Let us now turn to the specification by Díaz, Puch, and Guilló (2004). This framework is a bit different, though. In this framework there are two types of capital, working capital and energy saving capital. The energy efficiency of the economy, v , is given by the amount energy saving capital installed in the plant, which can be accumulated independently from working capital, k . Thus, capital services of one unit of working capital are

$$\kappa_t(u_t) = u_t \min \left\{ e_t(u_t), \frac{\zeta}{u_t} \right\}. \quad (3.6)$$

Plants' managers hire both types of capital every period before the energy price is known. The key assumption is that accumulating energy saving capital is subject to adjustment costs which implies

that energy use will respond sluggishly to energy prices. Appendix G describes this economy and its associated quasi-social planner's problem is

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + \iota_t \leq A_t \zeta^\alpha k_t^\alpha h_t^{1-\alpha} - \varrho_t e_t, \\
& c_t \geq 0, \ell_t \leq \bar{h} - h_t, \\
& e_{t+1} \geq \frac{\zeta(\iota_t - \psi(u_{t+1}k_{t+1}, u_t k_t))}{u_{t+1}(1+u_{t+1})} + (1 - \varpi) \frac{u_t(1+u_t)}{u_{t+1}(1+u_{t+1})} e_t, \\
& 0 \leq k_{t+1} \leq \frac{1}{1+u_{t+1}} \iota_t - \frac{\psi(u_{t+1}k_{t+1}, u_t k_t)}{1+u_{t+1}} + (1 - \varpi) \frac{1+u_t}{1+u_{t+1}} k_t, \\
& k_0, u_0, \text{ and energy prices given, } t \geq 0,
\end{aligned} \tag{3.7}$$

where the total amount of energy saving capital is $\mathbf{u}_t = u_t k_t$. Services of working capital are just $\kappa_t = \zeta k_t$. The total amount of investment, net of reallocation costs is $x_t = \frac{(\iota_t - \psi(u_{t+1}k_{t+1}, u_t k_t))}{u_{t+1}(1+u_{t+1})}$, which has to be allocated to working capital and energy saving capital. The amount of energy saving capital u_t plays the role of energy efficiency in our setup, $v_t = u_t^{-1}$. Notice that energy efficiency at the plant level is reversible but the existence of adjustment costs, $\frac{\psi(u_{t+1}k_{t+1}, u_t k_t)}{1+u_{t+1}}$, has an aggregate effect similar to irreversibility—it slows down the response of aggregate energy efficiency to changes in energy prices. Thus, the response of energy efficiency in this economy is similar to that of Atkeson and Kehoe (1999).

4 Calibration and estimation

In this section we describe the procedure used to calibrate our economy and the estimation of the shocks. We briefly discuss the data that we use. We take the U.S. to be our reference economy. We construct series for the energy price and energy consumption as well as economic aggregates for the period 1960-2008. Since we assume that all energy is imported in our model economy, we need to construct measures of value added, investment and the capital stock excluding, respectively, output, investment, and capital of energy-producing sectors. Our measure of value added includes the imputed services of consumer durable goods and government capital. Capital and aggregate investment are defined accordingly. To obtain an aggregate series on energy consumption for the US economy, we construct a constant-price measure of the consumption of coal, petroleum, natural gas and electricity by end-use sectors. Correspondingly, our aggregate energy price is the ratio of energy use measured in current prices to energy use measured in constant prices. A full explanation

of the sources and methods used in our data construction is given in Appendix A.

4.1 The energy price shock

Following Atkeson and Kehoe (1999), we estimate an ARMA(1,1) process for the aforementioned energy price deflator parameterized by

$$\log \varrho_{t+1} = (1 - \rho) \log \bar{\varrho} + \rho \log \varrho_t + \phi \epsilon_t + \epsilon_{t+1}, \quad (4.1)$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ and $\bar{\varrho}$ is the average energy price in the data. We find that $\rho \simeq 0.94$, $\phi \simeq 0.45$ for 1960-08.

4.2 The specification of the investment specific shocks

In this section we discuss the set of observations that we are going to use to specify a structure for the ISTC processes, Θ_t , Λ_t and Γ_t . First, we assume that Γ_t is constant. This is consistent with assuming that the energy price has no trend, in line with our estimated price shown in expression (4.1). Thus, we have left Θ_t , which affects the technology that transforms final good into investment goods, and Λ_t , which affects the amount of capital services yielded by one unit of new capital. The literature on growth or real business cycles that incorporates ISTC do not differentiate between its two sources—higher productivity in producing investment goods or higher quality of those goods. See for instance, Greenwood et al. (2000), or Fisher (2006), among others. This literature assumes that the quality adjusted relative price of capital goods falls due to ISTC and estimates a process to match the statistical properties of this price in the data. In our notation, this literature assumes a relative price equal to $\tilde{q}_t = \Theta_t^{-1} \Lambda_{t+1}^{-1}$ since it abstracts from energy requirement. In that setting, the more traditional approach in Greenwood et al. (2000) consists of estimating a process for \tilde{q}_t that features a deterministic trend and transitory shocks. After Fisher (2006), however, it is common to estimate a process that features a stochastic trend.

Differently from the standard theory, we need to distinguish both ISTC sources. In order to do so, we need to select two model variables and two sets of observations. In our theory, the two ISTC shocks govern the evolution of the price of new investment, $q_t(t+1, v_{t+1}) = \Theta_t^{-1}$, and the quality-adjusted price of capital, $q_t^\kappa(t+1, v_{t+1}) = \Theta_t^{-1} \Lambda_{t+1}^{-1} v_{t+1}^{-1}$, as defined in (2.26). We chose as the

data counterpart of q_t the inverse of the ratio of the chain weighted NIPA deflator for non durable consumption and services over that of durable consumption expenditures and private investment in structures, which we denote as Q_t . We proceed in this way because the prices of durable expenditures and investment in structures have significant quality bias in NIPA (see Gort et al. 1999). Next, we use as the data counterpart of $q_t^\kappa(t+1, v_{t+1})$ the estimates of Gordon (1990) and Cummins and Violante (2002) (hereafter the GCV deflator) of the inverse of the quality-adjusted relative price of equipment and software, which we denote as Q_t^κ .⁷

Now we need to specify a stochastic structure for both ISTC processes. We build on Fisher (2006) and assume that the productivity in the production of capital goods, Θ_t , is non-stationary and its growth rate, θ , is subject to shocks, z_t^θ , whereas the quality level of new capital goods, Λ_{t+1} , exhibits deterministic growth at the rate λ and it is subject to transitory shocks, z_t^λ . Specifically:

$$\Theta_t = \Theta_{t-1} e^{\theta + z_t^\theta}, \quad (4.2)$$

$$\Lambda_{t+1} = (1 + \lambda)^t e^{z_t^\lambda}. \quad (4.3)$$

There are two reasons why we choose Λ_{t+1} to be subject to transitory instead of permanent shocks. In this way, the empirical model used to estimate both processes, (4.2), and (4.3), is stationary in first differences, as we will show in Section 4.4. There is another reason and it has to do with the fact that the model variable used to identify Λ_{t+1} , the quality-adjusted relative price of capital, $q_t^\kappa(t+1, v_{t+1})$, depends also on one non-observable variable, the efficiency type, v_{t+1} . In our identification strategy shown in Section 4.4 we use the fact that Λ_{t+1} is only subject to transitory shocks.

We are not the first in assuming two different shocks affecting Investment Specific Technical Change. Justiniano et al. (2011) also specify two investment shocks to study business cycle fluctuations in a neokeynesian model economy. There are significant differences, however, between their approach and ours. As in Justiniano et al. (2011) we assume one of the shocks is permanent, and it is the one that affects the productivity in the production of investment goods. These authors use the NIPA deflators to estimate this process, but they check the robustness of their results to the use of their update of the GCV deflator. They also assume the other investment shock to be transitory but they interpret it in a different manner. They call it a shock to the marginal efficiency

⁷Rodríguez-López and Torres (2012) extend the GCV prices by using the methodology proposed by Cummins and Violante (2002). We use this update.

of investment (MEI hereafter) and estimate it using a proxy for the market value of firms. We could think of our transitory shock to quality of new capital goods as a sort of MEI shock, although making the mapping is difficult since Justiniano et al. (2011) abstract from energy requirements.

4.3 The stationary version of our model economy

It is convenient to obtain a stationary representation of our model economy by normalizing with respect to the state of technology. In our economy, output and capital grow at different rates. Moreover, since one of the ISTC shocks is permanent, our economy features a stochastic trend.

Proposition 3. *Suppose that Assumption 1 holds, so that in equilibrium all installed capital is utilized, $e_t(z, v) > 0$, for all t . Suppose that the energy price has no trend, that $\Gamma_t = 1$, for all t , and that Θ_t and Λ_t are given by (4.2) and (4.3), respectively. The efficiency type v_{t+1} has a stochastic trend given by*

$$\Psi_t = \Theta_t^{\frac{-1}{\mu}}, \text{ for all } t. \quad (4.4)$$

Gross output, y_t , investment and consumption, x_t , c_t , and $p_t e_t$, have a trend X_t which satisfies

$$X_t = A_{t-1}^{\frac{1}{1-\alpha}} [\Lambda_t \Theta_{t-1}]^{\frac{\alpha}{1-\alpha}} \Theta_{t-1}^{\frac{-\alpha}{\mu(1-\alpha)}}. \quad (4.5)$$

The trend of capital and capital services satisfy

$$T_{t-1} = X_t \Theta_{t-1}, \quad T_{t-1}^\kappa = \Lambda_t \Psi_{t-1} T_{t-1}. \quad (4.6)$$

The trend of the relative price of capital and the trend of the relative price of capital services are, respectively:

$$T_t^q = \Theta_{t-1}^{-1}, \quad T_t^{q^\kappa} = T_t^q \Lambda_t^{-1} \Psi_{t-1}. \quad (4.7)$$

Proof. Taking into account (2.17) and that gross output and energy expenditure should grow at the same rate the result follows. \square

In our economy, capital —adjusted and not adjusted by quality— grows at a higher rate than

output. The key difference with any other standard two sector model economy with ISTC (see, for instance, Greenwood et al. 1997, or Fisher 2006) is the explicit modeling of energy demand. In our theory, energy efficiency of new capital, measured by v_{t+1} has a stochastic trend given by the stochastic growth of Θ_t , the productivity of the technology to produce capital goods. Higher Θ_t implies that the economy becomes more capital intensive. For the energy share to remain constant, energy efficiency should increase. Notice that changes in Λ_t do not affect the trend in energy efficiency, v_{t+1} . Its innovations, however, will affect the fluctuations in energy efficiency over time. Finally, notice that the trend in the relative price of capital is given by the permanent shock Θ_t , whereas the quality-adjusted price depends also on the trend of embodied technical change and that of energy efficiency.

4.4 Empirical strategy

Figure 2(a) shows the NIPA deflator for private investment in structures and durable consumption expenditures (hereafter, St&D) and the GCV deflator for investment in equipment and software (hereafter E&S), over non-durable consumption and services. The former (NIPA St&D) is used as a proxy for the price of new investment goods in our model economy, $q_t = \Theta_t^{-1}$, whereas the latter (GCV E&S) proxies the relative price of capital services, $q_t^\kappa = \Theta_t^{-1} \Lambda_{t+1}^{-1} v_{t+1}^{-1}$. We use these two prices to estimate a joint process defined over Q_t and Q_t^κ as discussed in Section 4.2 above:

$$Q_t = Q_{t-1} e^{\theta + z_t^\theta}, \quad (4.8)$$

$$Q_t^\kappa = Q_t (1 + \lambda)^t e^{z_t^{\tilde{\lambda}}}, \quad (4.9)$$

with $e^{z_t^{\tilde{\lambda}}} \equiv e^{z_t^\lambda} v_{t+1}$ since we cannot, a priori, disentangle the shocks in Λ_{t+1} from the endogenous response of energy efficiency v_{t+1} .

A stationary representation of the joint investment specific technical change processes can be described in a state space form according to

$$\xi_{t+1} = \Gamma + F\xi_t + \varepsilon_{z_{t+1}}, \quad \text{state equation}, \quad (4.10)$$

$$y_t = H\xi_t(+ Ax_t) + \nu_t, \quad \text{observation equation}. \quad (4.11)$$

where $y_t = [\nabla \log Q_t^\kappa \quad \nabla \log Q_t]'$, $\varepsilon_{z_t^m} = (1 - \rho_m L) z_t^m$, with $m = \tilde{\lambda}, \theta$, and Γ, F, H , and A are

matrices of parameters. The term $(+Ax_t) + e_t$ accounts for measurement error with exogenous variables x_t if necessary. The innovations ν_t are generated recursively using the Kalman filter, and they are used to form the sample log likelihood function for observables y_t . Parameters of interest are estimated by maximising numerically this likelihood, and those estimates are used to identify the ISTC shocks. The investment specific transitory shock, $\varepsilon_{z\tilde{\lambda}}$ is identified with the GCV E&S deflator, whereas the permanent shock $\varepsilon_{z\theta}$ is identified with the NIPA St&D deflator. The unobservable process is $\tilde{\Lambda}_{t+1} \equiv \Lambda_{t+1}v_{t+1}$, and consequently $Q_t^\kappa = Q_t\tilde{\Lambda}_{t+1}$. With unit matrix H , the empirical model can be then written:

$$\begin{bmatrix} \nabla \log Q_t^\kappa - (\lambda + \theta) \\ \nabla \log Q_t - \theta \end{bmatrix} = \begin{bmatrix} \rho_\lambda & \rho_\theta - \rho_\lambda \\ 0 & \rho_\theta \end{bmatrix} \begin{bmatrix} \nabla \log Q_{t-1}^\kappa - (\lambda + \theta) \\ \nabla \log Q_{t-1} - \theta \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{z\tilde{\lambda}} \\ \varepsilon_{z\theta} \end{bmatrix} \quad (4.12)$$

Thus, z_t^θ in (4.8) represents exactly the state $\nabla \log Q_t - \theta \equiv S_t$. However, combined states S_t and $S_t^\kappa \equiv \nabla \log Q_{t-1}^\kappa - (\lambda + \theta)$, represent $z_t^\theta + z_t^{\tilde{\lambda}} - z_{t-1}^{\tilde{\lambda}} \equiv \rho_\theta S_{t-1} + \varepsilon_{z\theta} + \rho_\lambda S_{t-1}^\kappa + \varepsilon_{z\tilde{\lambda}} - \rho_\lambda S_{t-1}$, where $z_t^{\tilde{\lambda}} \equiv z_t^\lambda + \log(v_{t+1})$ so we keep track of the bias in our estimation through the response of endogenous variable v_{t+1} to the shocks. To identify the embodied technological change shock, z_t^λ , we simulate our model economy feeding to it the innovations z_t^θ and $z_t^{\tilde{\lambda}}$ (as well as the energy price innovations). Then we obtain a path for energy efficiency v_{t+1} and calculate the innovation $z_t^\lambda = z_t^{\tilde{\lambda}} - \log(v_{t+1})$. We simulate again our model economy feeding to it z_t^λ until the equilibrium price of quality-adjusted capital in the model converges. We proceed in this way because quality innovations z_t^λ only affect the fluctuations of energy efficiency, v_{t+1} , but not its trend, since we have assumed Λ_{t+1} to be subject only to transitory shocks. Consequently, the trend of the quality-adjusted price is not affected by quality innovations.

Table 2 in its lower rows reports the parameter estimates obtained by using annual data over both the benchmark sample period 1959-2009, and also 1947-2009. As a robustness check, we also compute the estimates corresponding to quarterly data from 1959QIV to 2009QI and from 1947QIV to 2009QI. It can be checked that all parameter estimates are in line with those reported in the literature (Fisher 2006 or Justiniano et al. 2011, among others). For instance, σ_θ is about 0.5% quarterly, and say, $\sigma_{\lambda\theta}$ is about 6%, whereas ρ_θ is about 0.2 and $\rho_{\lambda\theta}$ is about 0.7.

Figure 2(b) shows the correlation of the innovations z_t^θ and z_t^λ to the Q_t and Q_t^κ processes. To

compute this correlation a window of 5 years (20 quarters, quarterly estimation) has been used. The vertical red lines correspond to years 1974, 1978 and 1992. The correlation lies above average (zero in the figure) right after the oil price shocks in the 1980s and the early 2000s, when the 2003-08 oil price shock first hits. On the contrary, the correlation lies below average during the late 1990s. This picture suggests that ISTC innovations seem to be related to energy price changes but during the 1990s.

Before showing our quantitative experiments we discuss the aggregate targets and the discipline imposed by our theory to calibrate the model.

4.5 Aggregate targets

The calibration of the model closely follows the methods discussed in Atkeson and Kehoe (1999) and Díaz et al. (2004). In this version of our benchmark economy we abstract from labor choice, and concentrate on the behavior of investment. Parameter values are calibrated so that selected statistics of the steady state of our economy match their counterparts in the data. Table 1 shows the values of the calibrated parameters. In our data, the share of labor income over Value Added is about 60 percent. This implies a value for $\alpha = 0.4406$. Investment is 28.41 percent of gross output, which implies a depreciation rate $\varpi = 0.0523$. We have two other technological parameters: the stationary value for energy intensity, $\tilde{v}_{t+1}^\mu \equiv v_{t+1}^\mu / \Theta_t$, which determines the capital-energy ratio at the steady state, and μ , which governs the dynamic response of energy use to shocks. The reason why we need to set the steady state ratio \tilde{v}_{t+1} is because of the complementarity of capital and energy at the plant level. Our theory accounts for the dynamics of such complementarity, but we need to set its steady state level. Hence, the ratio v_{t+1}^μ / Θ_t is set so that the aggregate share of energy in our economy, in absence of shocks, is constant and equal to the mean of the considered period. Finally, we have left to calibrate μ . As we show in Appendix D, the share of energy in gross output for the latest vintage is equal to α / μ , which should be lower than the share for any older vintage. As a matter of fact we show in Appendix D that

$$\frac{\varrho_t e_t(z, v_z)}{y_t(z, v_z)} = \frac{\alpha}{\mu} \frac{\Lambda_t \Gamma_t v_t^{1-\mu}}{\Lambda_z \Gamma_z v_z^{1-\mu}}, \text{ for all } z \leq t. \quad (4.13)$$

Thus we can use our theory to find an appropriate value for μ consistent with the aggregate share of energy being constant and equal to its counterpart in the data. This value is $\mu = 16.2409$.

5 Quantitative experiments

In this section we show the main experiments conducted. To study the dynamics of aggregate energy use and energy efficiency in our model economy we proceed in the following way: we feed into the model the energy price and the two ISTC shocks, estimated in section 4.4, consistent with the fact that the model delivers, endogenously, the path for the two relative prices of capital (the price of structures and non durable goods and the GCV relative price) observed in the US economy for the period 1990-2008. In this way we can measure the contribution of energy efficiency versus ISTC in the dynamics of all macroeconomic aggregates.

5.1 The data

We need to select the particular statistics that describe the dynamics of energy use and efficiency appropriately in the data to compare them with their counterparts in our model economy. In Figure 3(a), we plot the logarithm of the share of energy, pE/VA , the relative price of energy, p , and energy use, E/VA , for the US economy over the period 1960-2010.⁸ The share of energy has a very high volatility, in spite of begin small on average, 4.75 percent of our measure of gross output for the whole period 1960-2010, peaking 8.89 percent in 1981 and reaching a minimum of 3.18 percent in 1998.⁹ Let us turn to inspect our measure of energy price, p . The first thing that we notice is that the price tracks very closely the evolution of the energy share. The second thing is the size of the shocks experienced in the years 1974, 1979-81, and 2003-08 that dwarf other price shocks.¹⁰

Let us turn to inspect the evolution of energy use, the ratio E/VA . As we can see, this ratio is falling since the early 1970s. This behavior, though, is a composite effect of changes in energy consumption, E , and value added, VA . Figure 3(c) shows that energy consumption responds belatedly to changes in energy prices but it has had a different behavior since the 1990s. In particular, it fell during the late 1970s, when value added was stagnant and has grown at a slower pace than value added since 1990. Thus, the ratio E/VA was more or less constant during the late 1980s and fell

⁸All aggregates have also been deflated using the implicit price deflator of non durable consumption goods and services.

⁹It may be thought that the evolution of this share hides significant differences in prices for the different energy sources considered. This is not the case; as pointed out by Kilian (2008), the pattern of fluctuations in the share of energy primarily reflects changes in the price of crude oil rather than shifts in energy consumption.

¹⁰More than 50 percent of all energy included in our measure E is petroleum-based for the entire period 1960-2010. That is, the important source of fluctuations in our measure of the energy price is the price of oil and its importance in our price index has not changed significantly during the period considered.

during the 1990s, in spite of the falling energy prices during that decade, and kept doing so during the 2000s when we witnessed the last upsurge in the energy price.¹¹

Now we turn to inspect Figure 3(b), where we show the capital to value added ratio, K/VA , and the capital to energy ratio, K/E . Capital is measured in units of value added. Notice that the ratio of both figures, $(K/VA)/(K/E)$, is equal to energy use, E/VA . The capital to value added ratio measures capital intensity and the capital to energy ratio is our aggregate measure of energy efficiency. Notice that the capital to value added ratio fluctuated less than the capital to energy ratio. This implies that the observed changes in energy use were mostly due to changes in aggregate energy efficiency, as measured by K/E . This ratio responds with some sluggishness to energy prices. When the price increases, the capital to energy ratio falls. This was the case for all years but for the 1990s, where the ratio decreased in spite of the falling energy price. It is also interesting to note that the capital to value added ratio decreased whereas the capital to energy ratio increased during the 1990s. This suggests that capital was becoming more productive and more efficient at the same time during that decade.

5.2 The dynamics of energy use and the capital-energy ratio

Figures 4(a) and 4(b) show, respectively, the evolution of energy use, the energy share and the evolution of our aggregate measure of efficiency, the capital to energy ratio. In this Figure capital is measured in units of value added, in the model as well as in the data. Figure 4(a) shows that our theory accounts remarkably well for the evolution of energy use in the US throughout the period 1960-2008. This is also why the energy share mimics so well its counterpart in the data.

In order to assess the importance of ISTC shocks in our theory of investment and energy use we strip our economy from them. This alternative economy is essentially the economy studied by Atkeson and Kehoe (1999) calibrated to reproduce the aggregate targets shown in Table 1. We should note that there is no steady state growth in the Atkeson and Kehoe (1999) economy, whereas our economy features a stochastic trend. Thus, we compare the detrended version of our economy with theirs. In this way we compare one economy where there are energy price shocks and the relative price of investment is always one (AK economy hereafter) and another economy with

¹¹It may be argued that the behavior of our measure of energy use is affected by the choice of base year prices. For instance, Metcalf (2008) measures energy consumption as the ratio of the amount of BTUs yielded by various energy sources to measured GDP. The behavior of his measure of energy use is fairly similar to the evolution of our measure.

energy price shocks and shocks to the relative price of capital services (our benchmark economy).

5.3 The role of ISTC shocks

The simulated evolution of energy use is shown in Figure 4(c). In the AK economy, energy use reacts to prices more than in the benchmark economy. Moreover, it rises since mid 1980s until the energy price starts rising again in the 2000s, missing completely the change of regime observed during the 1990s. As for the capital to energy ratio, shown in Figure 4(d), two features of this ratio stand out: first, its evolution is smoother than the evolution of its counterparts in the data and in our benchmark economy, respectively, and, second, it also misses the change of regime during the 1990s.

To understand further the role of ISTC shocks we need to recall that labor is inelastically supplied and all capital is always utilized in equilibrium in both model economies. Thus, after receiving a shock, both economies respond by changing the level of investment and the energy efficiency built into that investment. Figure 4(e) shows the evolution of the capital to value added ratio in both economies. As we can see, this ratio is more responsive to energy price shocks in our benchmark economy. This is so because investment is more volatile in the presence of ISTC shocks. The AK economy also misses the fall in this ratio observed since the mid 1980s until the 1990s, which is captured by our benchmark economy. Let us turn to inspect Figure 4(f) which compares the level of efficiency built into new capital goods in our benchmark economy (red line) and the AK economy (blue line). We are plotting here the detrended value of v_{t+1}^{-1} (multiplied by $\Theta_t^{\frac{1}{\mu}}$) with its counterpart in the AK economy. Notice that energy efficiency is also less volatile in the AK economy than in our benchmark economy. During the 1970s and early 1980s, our benchmark economy invests in more efficient machines than the AK economy. This is so because ISTC growth is only realized by investing, which rises the energy demand of the economy. If expected prices are high, the only way to reduce the future energy bill is by investing in efficient machines. On the contrary, during the 1990s, a period of high embodied ISTC growth and low energy prices, our benchmark economy invest less in efficiency than the AK economy and produces an investment boom. This is so because agents try to materialize all the benefits of transitory embodied ISTC shocks. In spite of the fall in efficiency during the 1990s, energy use in our benchmark economy falls because the accelerated ISTC embodied in new capital goods renders them more productive without rising energy consumption.

5.4 Permanent versus transitory ISTC shocks

We have distinguished between a permanent shock to the production of investment goods, Θ_t , and a transitory shock to the technical level embodied in the new capital good, Λ_t . Thus, we have specified that ISTC allows to produce not only more machines but also better machines at the same cost in units of final good. Typically, the literature on ISTC growth abstracts from such a distinction. In our case, this decomposition is important because the two shocks have different effects on the demand of energy and, therefore, in the level of efficiency built into new capital goods. In this section we would like to assess the contribution of each shock separately.

We start by shutting down the transitory shock and simulate again our economy. Basically, the permanent ISTC shock is needed to capture the trend in energy use and the capital to energy ratio observed in the data, as shown in Figure 5(c), specially after the 1990s. Permanent shocks alone, however, cannot capture the fluctuations in the capital to energy ratio. This is mirrored by the lower volatility of efficiency of new capital goods, as shown in Figure 5(e). Now we shut the permanent shock and leave the transitory shock only. This economy cannot capture the change of regime observed in 1990s; as we can see in Figure 5(b), energy use has no trend after 1990s, whereas it has a negative trend in the data. The implication of this behavior of energy use is that the capital to energy ratio has no trend either, as shown in Figure 5(d).

Summarizing, permanent shocks are needed to capture the trends observed in energy use and the capital to energy ratio, whereas the transitory shocks are mostly needed to capture the fluctuations in the capital to energy ratio. Fluctuations in energy efficiency are affected by both permanent and transitory ISTC shocks. Figures 5(e) and 5(f), however, confirm our guess: Embodied technical change (the one we have assumed to be transitory) is a sort of energy saving technical change. In particular, during the 1990s investment in efficiency would have been higher had embodied technical change not accelerated. It is interesting to note, though, that the evolution of the capital to value added ratio is very similar in both economies (when capital is measured in units of output), as shown in Figures 5(g) and 5(h). This implies that in order to disentangle permanent from transitory ISTC shocks we need further information that the one provided by the capital to value added ratio. Hence, our theory of investment and energy use gives us guidance about how to identify both ISTC shocks.

5.5 The short run price elasticity of energy use

Finally, it has been argued that the relationship between energy prices and the macroeconomy seems to have changed in the last two decades, since the response of GDP to the 2003-08 oil-price shock has been much softer than in the 1970s and 1980s (see, for instance, Kilian 2008, or Blanchard and Galí 2007). On this respect, Baumeister and Peersman (2012) find evidence of a decrease in the short run price elasticity of aggregate energy demand. This is corroborated by microeconomic evidence that suggests that the short run price elasticity of energy demand to energy prices may have changed and, in fact, decreased and that this change is due to the fact that capital has become more energy efficient over time; see Edelstein and Kilian (2007), Metcalf (2008), or Steinbuks and Neuhoﬀ (2010).

Our theory tells us that these findings are related and key to understand the effects of the 2003-08 oil price shock. In our model economy, right after an unexpected price shock energy consumption, e_t , does not change since all capital installed is utilized in equilibrium. Value added, though, falls and, therefore, so does energy use (energy consumption per unit of value added, $e_t/(y_t - \varrho_t e_t)$). Appendix E shows that the value of the short run elasticity of energy use with respect to the price is equal to the share of energy in value added,

$$\varepsilon_{eu_t} = \frac{\varrho_t e_t}{y_t - \varrho_t e_t}. \quad (5.1)$$

Hence, according to our theory, by increasing investment in the years of high ISTC the economy was increasing the average efficiency of capital and lowering the response of energy use (and value added) to energy price shocks. This implication of our theory is shown in Figure 3(a), which depicts energy use and the energy share in the data. The energy price level in the peak years 1981 and 2008 is around 60 percent above the mean of the entire period. Yet, the energy share is significantly lower in 2008; 6.66 percent versus 8.89 in 1981. Thus, according to our theory, energy use has been less responsive to changes in energy prices during the last oil price shock because energy efficiency is much higher than in previous decades.

6 Concluding remarks

In this paper we propose a theory to study how technology shapes the response of energy demand to energy price changes. In our theory, aggregate demand of energy depends on the complementarity of capital and energy at the plant level and the state of investment specific technical change (ISTC). The complementarity is modeled as an energy requirement built into irreversible capital, whereas ISTC implies that capital of different vintages differ in productivity due to technical progress. Thus, capital is heterogenous in two dimensions: its vintage and its energy requirement. A lower energy requirement (higher efficiency) comes at the cost of less capital services.

In our model, energy prices and the nature of investment specific technical change affect the choice of energy requirement built into new capital units. If ISTC reduces the cost of producing new capital goods, higher investment rises the exposure to energy price risk, which will entice investing in energy efficient, less productive, capital. If ISTC brings higher quality in capital, higher investment becomes more productive without providing an energy burden, which reduces the incentives to invest in energy efficiency. The complementarity of capital and energy at the plant level implies that the energy share in aggregate gross output varies with the distribution of capital across vintages and efficiency levels. We can, however, establish conditions under which there is a representation for aggregate gross output by aggregating capital using cost production prices.

Nevertheless, the complex interaction of ISTC and energy efficiency cannot be ascertained theoretically. This is why we study the dynamic properties of our theory quantitatively. To do so, we calibrate our model economy to match selected statistics of the US economy. Then, we focus on three shocks: one governing the energy price and two other governing the extensive and intensive margins of ISTC. The identification of these shocks relies on observables: the relative price of energy and the relative price of investment goods, not quality adjusted as well as quality adjusted. One of the points that we make in this paper is that the identification of ISTC shocks in existing literature reduces the scope of factors that influence the efficiency of producing investment goods relative to consumption goods. According to our theory, however, the relative price of investment goods depends not only on the two ISTC shocks but also on the energy efficiency built into capital units. Thus, the model imposes a novel discipline to disentangle the effects of ISTC shocks from the effects of energy efficiency. This discipline can be interpreted as a theoretical foundation to the view expressed by Gordon (1996), who argued that changes in energy efficiency is one of the key

determinants of the dynamics of the relative price of investment goods. Our theory also gives us a discipline for aggregating output and capital in a way that is consistent with National Accounts.

Our model with ISTC accounts for the fall of energy use per unit of output observed during the 1990s, a period in which energy prices fell below trend. This pattern of the data cannot be accounted for by previous theories of energy demand, which abstract from ISTC and assume that all technical change is neutral. According to previous theories, the fall in the energy price during the 1990s should have induced higher investment in less efficient capital and a rise in energy use per unit of output. According to our theory, the 1990s were a period of accelerated ISTC that raised quality of capital substantially. This rise in productivity allowed to lower energy efficiency of new capital goods and, at the same time, reduced energy use per unit of output at the aggregate level.

We have abstracted from the determinants of energy supply as well as from either environmental concerns or exhaustible energy. Although we have focused our analysis on annual fluctuations, we think that this theory can be used to address more longer term questions as climate change. We have also abstracted from taxes and the related issue of giving subsidies to using renewable energies. We leave these issues for further research.

Appendix

A The data

In this Appendix we document the construction of the data series we use in the empirical part of the paper. We obtain data from two sources: the Annual Energy Review (2000) and National Income and Product Accounts. The data we use can be accessed in the addresses: <http://www.eia.gov/> and <http://www.bea.gov>. From now on we will refer to each source as AER, and NIPA, respectively. Our data set is available upon request.

A.1 Output, consumption, investment, and the capital stock

We follow the method described by Cooley and Prescott (1995) to construct broad measures of output, consumption, investment, and the capital stock. Specifically, our measure of capital includes private stock of capital, the stock of inventories, the stock of consumer durable goods and the government stock. Consequently, the measured value of GDP is augmented with the imputed flow of services from the stock of durable goods and the government stock. We subtract from each of the series of output, investment and capital the corresponding series for the energy producing sectors: oil and gas extraction, electricity and gas services, petroleum and coal production, and pipeline transportation, as our theory cannot account for the behavior of the energy-producing sectors. We have information on the three variables for the last two sectors but about the first two sectors we only have information about the net stock of capital, and we use it to impute estimates of output and investment. Gross output is the sum of value added and the final expenditure on energy. Real variables are obtained by dividing the nominal variables by the implicit price deflator of non durable consumption goods and services.

A.2 Energy price, use, and expenditures series

Our energy data covers the primary energy consumption of end-users and is obtained from the Annual Energy Review (AER, hereafter). We consider four forms of energy: coal, petroleum, natural gas and electricity. AER (Table 2.1a) gives data on total energy consumption by end users measured in British thermal units (BTUs) disaggregated into the four forms of energy considered. We denote these data on energy consumption for each type of energy by Q_{it} , where the index i denotes the form of energy.

This measure Q_{it} is already net of energy consumption of the electricity sector. We subtract from total primary energy consumption of the industrial sector that of four energy sectors: oil and gas extraction, electricity and gas services, petroleum and coal production, and pipeline transportation. The BEA gives information on the net stock of Fixed Assets by industry and we assume that the amount of BTUs consumed by those four sectors, as a proportion of BTUs consumed by the industrial sector, is the same that the amount of capital in those sectors as a proportion of assets in the industrial sector.

We construct a constant-price measure of energy consumption. We choose the base year to be 2005 and define total energy use to be $E_t = \sum_i Q_{it} P_{i0}$, where P_{i0} is the price in dollars per BTUs of

energy type in 2005 from AER, divided by the implicit price deflator of non durable consumption goods and services in NIPA (which is constructed as a weighted average of the two implicit price deflators). For coal, natural gas and petroleum we use the production price series (AER, Table 3.1). For electricity, we use the retail price of electricity sold by electric utilities (see AER, Table 8.10). In Table 8.10 the price for electricity is in cents per kilowatt-hour. We use AER Table A.6 to convert the price to cents per BTUs. All prices are in real terms; i.e., divided by the implicit price deflator of non durable consumption goods and services. We construct the energy price deflator as

$$p_t = \frac{\sum_i Q_{it} P_{it}}{\sum_i Q_{it} P_{i0}}. \quad (\text{A.1})$$

Finally, energy expenditure is $p_t \cdot E_t = \sum_i Q_{it} P_{it}$.

B The quasi-social planner's problem

The efficient allocation for this economy can be found as the solution to the following planning problem:

$$\begin{aligned} \max_{\substack{c_t, x_t(t+1, v), \\ \ell_t, k_{t+1}(t+1, v)}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\ell_t)) \\ \text{s. t.} \quad & c_t + \int_0^{\infty} x_t(v) dv \leq \sum_{z=-\infty}^t \int_0^{\infty} (1-\varpi)^{t-z} k_z(z, v) [y_t(z, v) - \varrho_t e_t(z, v)] dv, \\ & 0 \leq y_t(z, v) \leq A_t \kappa_t(z, v)^{\alpha} h_t(z, v)^{1-\alpha}, \\ & 0 \leq \kappa_t(z, v) \leq \Lambda_z \Gamma_z v^{1-\mu} e_t(z, v), \\ & 0 \leq e_t(z, v) \leq \Gamma_z^{-1} v^{\mu}, \\ & 0 \leq \ell_t \leq \bar{h} - \sum_{z=-\infty}^t \int_0^{\infty} (1-\varpi)^{t-z} k_z(z, v) h_t(z, v) dv, \\ & 0 \leq h_t(z, v) \leq \bar{h}, \\ & 0 \leq k_{t+1}(t+1, v) \leq \Theta_t x_t(v), \quad t \geq 0, \\ & k_z(z, v) \text{ given, } \quad z \leq t, \quad v \geq 0. \end{aligned} \quad (\text{B.1})$$

C Decentralization of the efficient allocation

Here we show how to decentralize the economy described in Section 2.

C.1 Market arrangements

We assume that households are the owners of the plants and, therefore, of the capital installed. There is a market for plants that opens at the end of the period, once profits have been realized. Notice, though, that capital is not traded since it is already installed in a plant and it cannot be reallocated. Since there is a one to one correspondence between plants and units of capital, the price of a plant is also equal to the price of the unit of capital installed, $p_t(z, v)$, where $p_t(z, v)$ is the

price of one unit of capital of vintage z and type v at the end of period t in units of consumption good at time t . We further assume that all households start out with the same amount of shares of the plants installed. Additionally, we assume that households trade a one risk free bond which is in zero net supply.

The timing is the following: At the end of period $t - 1$ any prospective plant must install one unit of capital before the energy price is known. After this decision has been made, at the beginning of period t the uncertainty is resolved: agents learn the productivity of the investment technology Θ_t and the quality of new capital goods, Λ_{t+1} . The energy price is realized. Then, they decide the amount of energy to be consumed, $e_t(z, v)$, and the number of workers hired, $h_t(z, v)$. Households consume and save. A fraction ϖ of plants die.

C.1.1 The household's problem

Plants of any vintage and type can be traded at the individual level. New investment, however, comes in new vintage—it is a technological restriction, as in the one sector model TFP grows exogenously, we cannot help to be more productive. Agents can, though, choose the type of the new capital units to be installed. The household's problem can be written in the following way:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + \int_0^{\infty} x_t(v) dv + \sum_{z=-\infty}^{t+1} \int_0^{\infty} p_t(z, v) m_{t+1}(z, v) dv + b_{t+1} \leq w_t(\bar{h} - \ell_t) + (1 + r_t^b) b_t + \\
& \int_0^{\infty} p_t(t+1, v) k_{t+1}(t+1, v) dv + \sum_{z=-\infty}^t \int_0^{\infty} [(1 - \varpi) p_t(z, v) + \pi_t(z, v)] m_t(z, v) dv, \quad (\text{C.1}) \\
& k_{t+1}(t+1, v) \leq \Theta_t x_t(v), \quad v > 0, \\
& x_t \geq 0, \quad m_{t+1}(z, v) \geq 0, \quad k_{t+1}(t+1, v) \geq 0, \quad \text{for all } z \leq t+1, \quad v > 0, \\
& b_{t+1} \geq \underline{b}, \\
& m_0(z, v), \quad b_0, \quad \text{and energy prices given.}
\end{aligned}$$

The constraint $k_{t+1}(t+1, v) \leq \Theta_t x_t(v)$ implies that the amount of new capital that agents can sell in the market cannot be higher than the amount of good needed to create them.

C.1.2 The plant's problem

$$\begin{aligned}
\max_{\substack{y_t(z, v) \geq 0, h_t(z, v) \geq 0, \\ e_t(z, v) \geq 0}} \quad & \pi_t(z, v) = y_t(z, v) - w_t h_t(z, v) - \varrho_t e_t(z, v) \\
\text{s. t.} \quad & y_t(z, v) \leq A_t \kappa_t(z, v)^\alpha h_t(z, v)^{1-\alpha}, \\
& \kappa_t(z, v) \leq \Lambda_z \Gamma_z v^{1-\mu} \min\{e_t(z, v), \Gamma_z^{-1} v^\mu\}. \quad (\text{C.2})
\end{aligned}$$

C.1.3 Definition of equilibrium

An equilibrium for this economy, given the sequence of energy prices, $\{\varrho_t\}_{t=0}^{\infty}$, is a sequence of prices $\left\{ \{p_t(z, v)\}_{z=-\infty}^{t+1}, w_t, r_t^b \right\}_{t=0}^{\infty}$, an allocation $\left\{ c_t, \ell_t, \{m_t(z, v)\}_{z=-\infty}^{t+1}, x_t(v), k_{t+1}(t+1, v), b_{t+1} \right\}$

for each consumer, and an allocation for each plant of variety (z, v) , $\{y_t(z, v), h_t(z, v), \kappa_t(z, v), e_t(z, v)\}_{z=-\infty}^t$, $v \in \mathbf{R}_{++}$, such that:

1. $\{c_t, \ell_t, \{m_t(z, \cdot)\}_{z=-\infty}^{t+1}, x_t(v), b_{t+1}\}$ solves the household's problem shown in (C.1) given the sequence of prices,
2. $\{y_t(z, v), h_t(z, v), e_t(z, v)\}_{z=-\infty}^t$, $v \in \mathbf{R}_{++}$, solves the plant's problem given the sequence of prices,
3. the relative price of the latest vintage is $p_t(t+1, v) = \Theta_t^{-1}$, for any v ,
4. markets clear,
 - (a) the bond is in zero net supply, $b_{t+1} = 0$,
 - (b) the amount of plants of class (z, v) must be equal to the amount of capital of that class, $m_t(z, v) = k_t(z, v)$, for all $z \leq t$, $v > 0$,
 - (c) the labor market clears, $\bar{n} - \ell_t = \sum_{z=-\infty}^t \int_0^\infty m_t(z, v) h_t(z, v) dv$,
 - (d) the final good market satisfies $c_t + x_t = \sum_{z=-\infty}^t \int_0^\infty m_t(z, v) y_t(z, v) dv$,
5. and the law of motion of capital of class (z, v) is $k_t(z, v) = (1 - \omega)^{t-z} \Theta_{z-1} x_{z-1}(v)$, for all $t \geq z$, $v \in \mathbb{R}_+$, for all t .

C.1.4 Some properties of equilibrium

We have shown in Section 2.5 some properties of the efficient allocation. In this economy, given the energy prices, the Welfare Theorems hold; thus, in equilibrium Propositions 1 to 2, as well as Corollary 2, hold. Thus, we keep Assumption 1. Clearly, as shown in Proposition 2, the price of a new plant, $p_t(t+1, v_{t+1})$, is equal to the cost of producing the unit of capital installed in that plant, which was defined as $q_t(t+1, v_{t+1}) = \Theta_t^{-1}$. Moreover, we know, using Proposition 2 that agents only invest in one type of capital, that for which the present value of all future profits is maximized. The price of an old plant is equal to the present value of profits that the plant will accrue in the future:

$$p_t(z, v_z) = E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \frac{\varphi_{t+i}}{\varphi_t} [\alpha y_{t+i}(z, v_z) - \varrho_{t+i} \Gamma_z^{-1} v_z^\mu], \quad (\text{C.3})$$

Using (2.12) we can write (C.3) as

$$p_t(z, v_z) = \frac{\Lambda_z v_z}{\Lambda_{t+1} v_{t+1}} p_t(t+1, v_{t+1}) + \left(\frac{\Lambda_z v_z}{\Lambda_{t+1} v_{t+1}} - \frac{\Gamma_z^{-1} v_z^\mu}{\Gamma_{t+1}^{-1} v_{t+1}^\mu} \right) E_t \sum_{i=1}^{\infty} (1 - \varpi)^{i-1} \frac{\varphi_{t+i}}{\varphi_t} \varrho_{t+i} \Gamma_{t+1}^{-1} v_{t+1}^\mu. \quad (\text{C.4})$$

Since for the latest class of capital the present value of all energy expenditures is equal to the fraction α/μ of expected gross output, using (2.16) we find that

$$p_t(z, v_z) = \frac{\Lambda_z v_z}{\Lambda_{t+1} v_{t+1}} \Theta_t^{-1} + \left(\frac{\Lambda_z v_z}{\Lambda_{t+1} v_{t+1}} - \frac{\Gamma_z^{-1} v_z^\mu}{\Gamma_{t+1}^{-1} v_{t+1}^\mu} \right) \frac{\Theta_t^{-1}}{\mu - 1}. \quad (\text{C.5})$$

This price is different than the cost of unit of capital installed, as defined in Definition 1, since it takes into account the associated cost of energy. Actually if the ratio of capital services to energy expenditure is higher than in a new plant, then $p_t(z, v_z) > q_t(z, v_z)$; otherwise the cost valuation is higher than the market price of the plant, $p_t(z, v_z) < q_t(z, v_z)$.

D Calibration

Let us write the planner's problem in the following way:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\bar{h} - h_t)) \\
\text{s. t.} \quad & c_t + x_t \leq A_t (\Theta_{t-1} \Lambda_t v_t \tilde{\kappa}_t)^\alpha h_t^{1-\alpha} - \varrho_t e_t, \\
& e_{t+1} \geq \Gamma_{t+1}^{-1} v_{t+1}^\mu \Theta_t x_t + (1 - \varpi) e_t, \\
& 0 \leq \tilde{\kappa}_{t+1} \leq x_t + (1 - \varpi) \frac{v_t \Lambda_t \Theta_{t-1}}{v_{t+1} \Lambda_{t+1} \Theta_t} \tilde{\kappa}_t, \\
& \tilde{\kappa}_0 \text{ given, } x_t \geq 0.
\end{aligned} \tag{D.1}$$

$\tilde{\kappa}_t$ is the amount of capital services in units of gross output (i.e., multiplied by $q_{t-1}\kappa$). We calibrate α so that labor share in Value Added matches its corresponding counterpart in the data. Let us now turn to the law of motion of capital. Since capital is expressed in units of gross output, the ratio must be constant at the steady state,

$$\frac{y_{t+1}}{y_t} \frac{\tilde{\kappa}_{t+1}}{y_{t+1}} = \frac{x_t}{y_t} + (1 - \varpi) \frac{v_t \Lambda_t \Theta_{t-1}}{v_{t+1} \Lambda_{t+1} \Theta_t} \frac{\tilde{\kappa}_t}{y_t}. \tag{D.2}$$

Matching the capital-output ratio and the investment ratio yields

$$1 - \tilde{\varpi} \equiv (1 - \varpi) \frac{v_t \Lambda_t \Theta_{t-1}}{v_{t+1} \Lambda_{t+1} \Theta_t}. \tag{D.3}$$

Since $q_t^\kappa = \Theta_t^{-1} \Lambda_{t+1}^{-1} v_{t+1}^{-1}$, we can obtain

$$1 - \varpi = (1 - \tilde{\varpi}) \frac{q_{t-1}^\kappa}{q_t^\kappa}. \tag{D.4}$$

At the steady state, the share of energy, $\varrho_t e_t / y_t$, must be constant, as c_t / y_t and the investment ratio. Now we can rewrite the law of motion of energy as

$$\frac{\varrho_t}{\varrho_{t+1}} \frac{y_{t+1}}{y_t} \frac{\varrho_{t+1} e_{t+1}}{y_{t+1}} = \left(\frac{\Gamma_{t+1}}{\varrho_t} \right)^{-1} v_{t+1}^\mu \Theta_t \frac{x_t}{y_t} + (1 - \varpi) \frac{\varrho_t e_t}{y_t}. \tag{D.5}$$

Thus, matching the energy share determines the stationary value of \tilde{v}_t^μ ,

$$\tilde{v}_t^\mu = \left(\frac{\Gamma_{t+1}}{\varrho_t} \right)^{-1} v_{t+1}^\mu \Theta_t. \tag{D.6}$$

We have left obtaining μ . Unless we assume that there is no neutral progress, we cannot use the gross output growth rate to calibrate μ . For the same reason, we cannot use q_t^κ . We know that for any new vintage installed, energy expenditures are the share α/μ of gross output. If, at the steady state, the energy price is constant, $\varrho_t = \bar{\varrho}$, for all t , by using (2.16) it is easy to show that

$$\frac{\varrho_t e_t(t, v_t)}{y_t(t, v_t)} = \frac{\alpha}{\mu}, \text{ for all } t. \quad (\text{D.7})$$

Likewise,

$$\frac{\varrho_t e_t(z, v_z)}{y_t(z, v_z)} = \frac{\alpha}{\mu} \frac{\Lambda_t \Gamma_t v_t^{1-\mu}}{\Lambda_z \Gamma_z v_z^{1-\mu}}, \text{ for all } z \leq t. \quad (\text{D.8})$$

The energy share on aggregate gross output is

$$\frac{\varrho_t e_t}{y_t} = \sum_{z=-\infty}^t \frac{\varrho_t e_t(z, v_z)}{y_t(z, v_z)} \frac{y_t(z, v_z) (1-\varpi)^{t-z} \Theta_{z-1} x_{z-1}}{y_t}. \quad (\text{D.9})$$

Now, using our aggregation strategy and the fact the economy is at a steady state, we find that

$$\frac{\varrho_t e_t}{y_t} = \frac{\alpha}{\mu} \frac{\sum_{z=-\infty}^t \left(\frac{1-\varpi}{1+g_y} \right)^{t-z}}{\sum_{z=-\infty}^t \left(\frac{1-\varpi}{1+g_\kappa(\mu)} \right)^{t-z}}. \quad (\text{D.10})$$

Calling α^e to the share of energy in gross output in the data,

$$\alpha^e = \frac{\alpha}{\mu} \frac{\sum_{z=-\infty}^t \left(\frac{1-\varpi}{1+g_y} \right)^{t-z}}{\sum_{z=-\infty}^t \left(\frac{1-\varpi}{1+g_\kappa(\mu)} \right)^{t-z}}. \quad (\text{D.11})$$

This is a non-linear equation in μ , since it affects the growth rate of capital services, $g_\kappa(\mu)$. It must be that α^e is greater than α/μ , since older vintages operate with higher energy requirements than the new one. This is a key difference with Atkeson and Kehoe (1999).

E The short run elasticity of energy use

In our notation, energy use is

$$eu_t = \frac{e_t}{y_t - \varrho_t e_t}. \quad (\text{E.1})$$

Differentiating both sides with respect to ϱ_t , and multiplying by ϱ_t/eu_t we find

$$\frac{\partial eu_t}{\partial \varrho_t} \frac{\varrho_t}{eu_t} = \frac{\partial e_t}{\partial \varrho_t} \frac{\varrho_t}{e_t} + s_t^e - \frac{1-\alpha}{1-s_t^e} \frac{\partial h_t}{\partial \varrho_t} \frac{\varrho_t}{h_t}. \quad (\text{E.2})$$

s_t^e is the share of energy in value added. In our model economy energy consumption does not change after a price shock given our Assumption 1. Moreover, labor is inelastically supplied. Thus, the elasticity of energy use is

$$\varepsilon_{eu_t} = s_t^e. \quad (\text{E.3})$$

F The cost of saving energy in a putty-clay model economy

Atkeson and Kehoe (1999) abstract from investment specific technological progress but retain the assumption about efficiency types and capital irreversibility. In particular, the amount of capital services, $\kappa_t(u)$, depends on the amount of energy used in the plant, $e_t(u) \geq 0$, according to the technology

$$\kappa_t(u) = f(u) \min \left\{ e_t(u), \frac{1}{u} \right\}, \quad (\text{F.1})$$

where $f(u)$ is a strictly increasing function of v , where $f'(u) \geq 0$, and $f''(u) < 0$. In this economy, the efficiency type u plays the same role that v^{-1} in our model economy. The production of one new unit of capital always takes one unit of output, which is equivalent to assuming in our framework that $\Theta_t = 1$, for all t .

The firms optimally choose $e_t(u) = u^{-1}$ and $\kappa_t(u) = f(u)/u$. By denoting $v = f(u)/u$, it is easy to show that $e_t(v)$ is an increasing and convex function of v . Thus, the social planner's problem in this case is exactly the one shown in expression (2.35). In this economy, the capital-energy ratio is solely governed by changes in capital type, v , which respond to changes in energy prices.

G A Putty-Putty model of energy with costly capital reallocation

This is a simplified version of Díaz, Puch, and Guilló (2004) in our environment.

G.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\ell_t)), \quad \beta \in (0, 1), \quad \xi > 0, \quad (\text{G.1})$$

where c_t is consumption and ℓ_t is leisure t . Each household is endowed with \bar{h} units of time and, therefore, works $\bar{h} - \ell_t$ hours every period.

G.2 Technology

Production of the unique final good is carried out at a continuum of autonomous plants which are indexed by the amount of energy-saving capital used, v . In each plant output is produced with

labor, energy and the unit of working capital installed, according to the technology

$$y_t(v) = A_t \kappa_t(v)^\alpha h_t(v)^{1-\alpha}, \quad (\text{G.2})$$

with $0 < \alpha < 1$, where A_t is the growth factor of the disembodied technological knowledge, $\kappa_t(v)$ is the amount of services provided by the unit of working capital installed, whereas $h_t(v)$ is the amount of labor services employed in the plant. The amount of services yielded by the unit of working capital, $\kappa_t(v)$, depends on the amount of energy used in the plant, $e_t(v)$, and the amount of energy-saving capital installed, v , according to the technology

$$\kappa_t(v) = v \min \left\{ e_t(v), \frac{\zeta}{v} \right\}. \quad (\text{G.3})$$

Each period households save and have the possibility of transforming final good into new units of working capital or new units of energy-saving capital. Investing in energy-saving capital, though, is subject to adjustment costs, which imply that working capital cannot be transformed on a one-to-one basis into energy-saving capital, and vice versa. Households rent out energy-saving capital to plants in period $t - 1$ to be used in period t . Plants can be scrapped at no cost. Finally, at the end of the period, once production has taken place, the unit of working capital installed has a positive probability of death, $\varpi \in [0, 1]$, which is i.i.d. across types and plants. This death probability plays the role of physical depreciation of working capital. To simplify the exposition of the model, energy-saving capital depreciates at the same rate $\varpi \in [0, 1]$.

G.3 Planner's problem

Notice that since plants can be scrapped at no cost, and the amount of energy-saving capital can be changed every period, all plants are ex-ante identical at all periods. Moreover, the total number of plants is always equal to the amount of working capital, k_t . Thus, the problem of a household is

$$\begin{aligned} \max_{\substack{c_t, x_t(v), \\ \ell, k_{t+1}(v)}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \varphi \log(\ell_t)) \\ \text{s. t.} \quad & c_t + x_t \leq \int_0^{\infty} k_t(v) [y_t(v) - \varrho_t e_t(v)] dv, \\ & 0 \leq y_t(v) \leq A_t \kappa_t(v)^\alpha h_t(v)^{1-\alpha}, \\ & 0 \leq \kappa_t(v) \leq v e_t(v), \\ & 0 \leq e_t(v) \leq \frac{\zeta}{v}, \\ & 0 \leq \ell_t \leq 1 - \int_0^{\infty} k_t(v) h_t(v) dv, \\ & 0 \leq h_t(v) \leq 1, \\ & k_{t+1} + \mathbf{v}_{t+1} - (1 - \varpi)(k_t + \mathbf{v}_t) + \psi(\mathbf{v}_{t+1}, \mathbf{v}_t) \leq x_t, \\ & k_{t+1}(v) \geq 0, \\ & \int_0^{\infty} k_{t+1}(v) dv \leq k_{t+1}, \\ & \mathbf{v}_{t+1} \geq \int_0^{\infty} v k_{t+1}(v) dv, \\ & k_0(v) \text{ given, } v \geq 0. \end{aligned} \quad (\text{G.4})$$

G.4 Properties of the efficient allocation

Proposition App. 1. *The ratio labor to working capital services is the same across all classes of working capital used:*

$$(1 - \alpha) A_t \kappa_t(v)^\alpha h_t(v)^{-\alpha} = \frac{u_\ell(c_t, \ell_t)}{u_c(c_t, \ell_t)}. \quad (\text{G.5})$$

Now we turn to investigate which classes are allocated energy.

Proposition App. 2. *If a time t the type $v > 0$, is allocated energy, $e_t(v) > 0$, then it must be the case that $e_t(v) = \zeta v^{-1}$ and $\kappa_t(v) = \zeta$.*

The previous Proposition implies a rule for the utilization of capital.

Corollary App. 1. *Only installed working capital of types $v \geq \underline{v}_t$ are utilized in equilibrium, where \underline{v}_t is defined as*

$$\alpha A_t^{\frac{1}{\alpha}} \left(\frac{1 - \alpha}{w_t} \right)^{\frac{1 - \alpha}{\alpha}} v = \varrho_t, \quad (\text{G.6})$$

and $w_t \equiv \frac{u_\ell(c_t, \ell_t)}{u_c(c_t, \ell_t)}$. The type \underline{v}_t increases with ϱ_t .

Let us turn now to analyze the investment decision and the characteristics of the new plants.

Proposition App. 3. *Working capital is installed to one efficiency type $v_{t+1} > 0$, $k_{t+1}(v_{t+1}) > 0$.*

Proof. The first order condition with respect to $k_{t+1}(v)$ is

$$\Psi_t^k + v \Psi_t^\mathbf{v} = E_t \lambda_{t+1} [\alpha y_{t+1}(v) - \varrho_{t+1} e_{t+1}(v)] + \Psi_t^{k_0}(v), \quad (\text{G.7})$$

where Ψ_t^k is the first order condition with respect the aggregate k_{t+1} , $\Psi_t^\mathbf{v}$ is the first order condition with respect to \mathbf{v}_{t+1} , $\Psi_t^{k_0}(v)$ is the multiplier associated to the non-negativity constraint on $k_{t+1}(v)$, and λ_{t+1} is marginal utility of consumption at time $t + 1$. Using Proposition App. 2 we can write the previous expression as

$$\Psi_t^k + v \Psi_t^\mathbf{v} = E_t \lambda_{t+1} \left[\alpha A_{t+1}^{\frac{1}{\alpha}} \left(\frac{1 - \alpha}{w_{t+1}} \right)^{\frac{1 - \alpha}{\alpha}} \zeta - \varrho_{t+1} \zeta v^{-1} \right] + \Psi_t^{k_0}(v), \quad (\text{G.8})$$

If $k_{t+1}(v) > 0$, then $\Psi_t^{k_0}(v) = 0$. Continuity of both sides of (G.8) with respect to v imply that only one type is used in equilibrium. Since working capital can be reallocated across types, only a particular type v_{t+1} is used. \square

G.5 Aggregation

In this economy, at any period t only one efficiency type is used at every period. Thus, all plants are alike and aggregate output is $y_t = y_t(v_t) k_t$; likewise happens to labor. Moreover, services of

working capital are just $\kappa_t = \zeta k_t$. The total amount of capital is $k_t + \mathbf{v}_t = (1 + v_t) k_t$. The amount of energy used every period is $e_t = \zeta k_t / v_t$. Thus, we can write the planner's problem as

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) + \xi \log(\ell_t)) \\
\text{s. t.} \quad & c_t + x_t \leq A_t \zeta^\alpha k_t^\alpha h_t^{1-\alpha} - \varrho_t e_t, \\
& c_t \geq 0, \ell_t \leq 1 - h_t, \\
& e_{t+1} \geq \frac{\zeta(x_t - \psi(v_{t+1}k_{t+1}, v_t k_t))}{v_{t+1}(1+v_{t+1})} x_t + (1 - \varpi) \frac{v_t(1+v_t)}{v_{t+1}(1+v_{t+1})} e_t, \\
& 0 \leq k_{t+1} \leq \frac{1}{1+v_{t+1}} x_t - \frac{\psi(v_{t+1}k_{t+1}, v_t k_t)}{1+v_{t+1}} + (1 - \varpi) \frac{1+v_t}{1+v_{t+1}} k_t, \\
& k_0, v_0, \text{ and energy prices given, } t \geq 0.
\end{aligned} \tag{G.9}$$

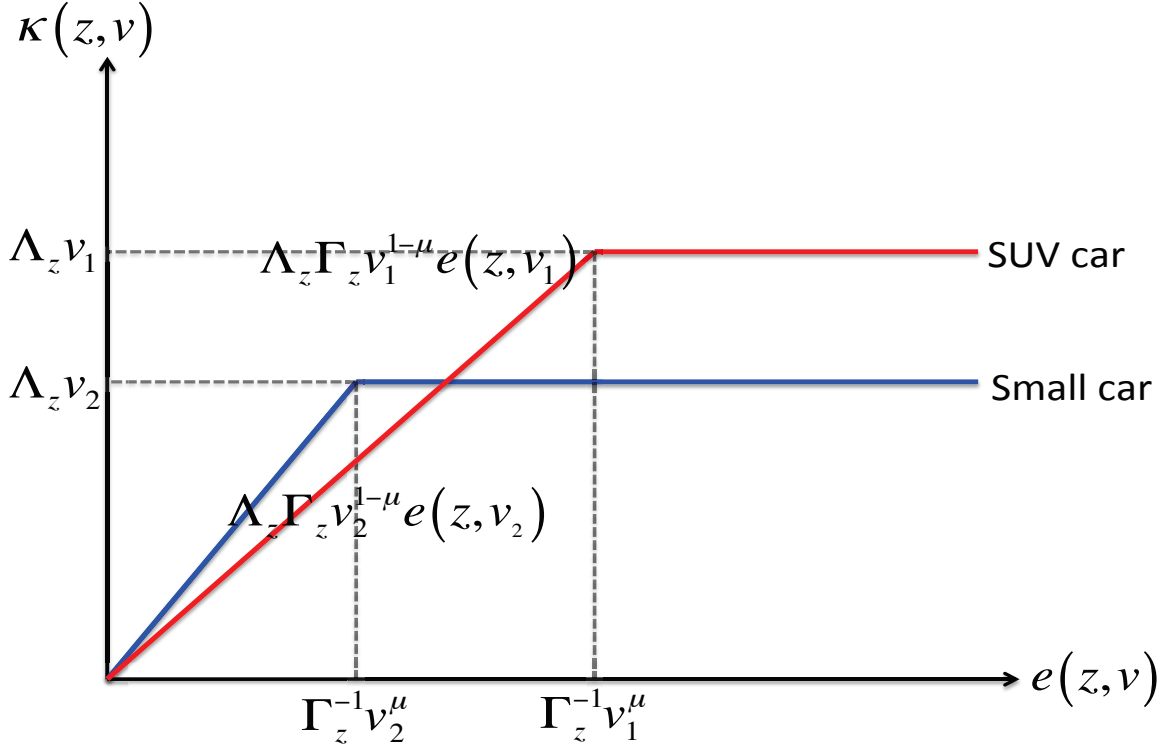
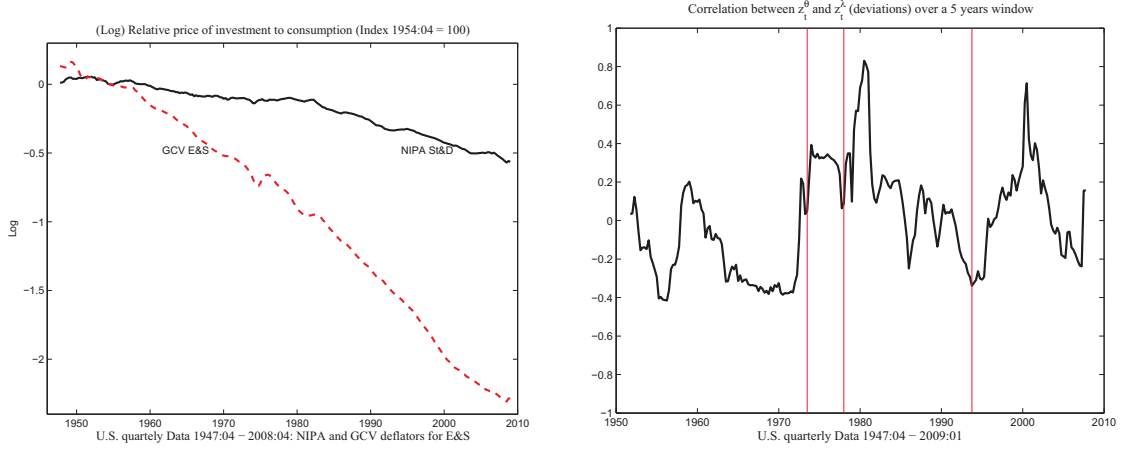


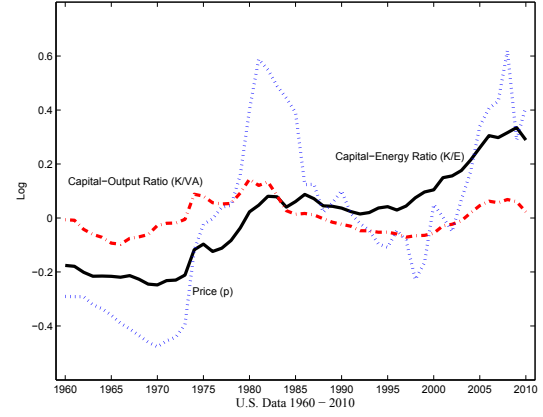
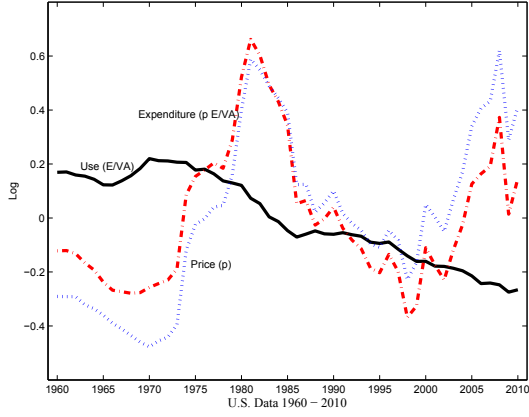
Figure 1: Energy requirement.



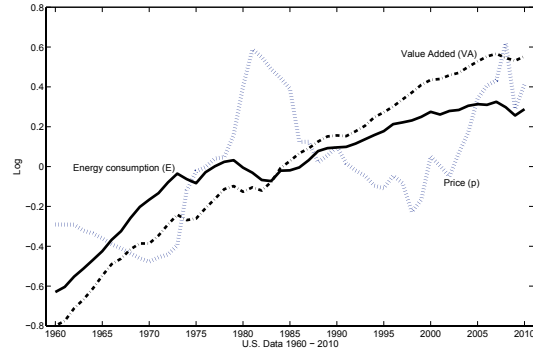
(a) Alternative measures of the relative price of investment. NIPA St&D is based on the National Income and Product Accounts deflators for structures and durables. GCV E&S uses the Gordon-Cummins-Violante deflator for equipment and software.

(b) A measurement of the joint evolution of the innovations to investment specific disturbances.

Figure 2: Investment prices and correlation of innovations.

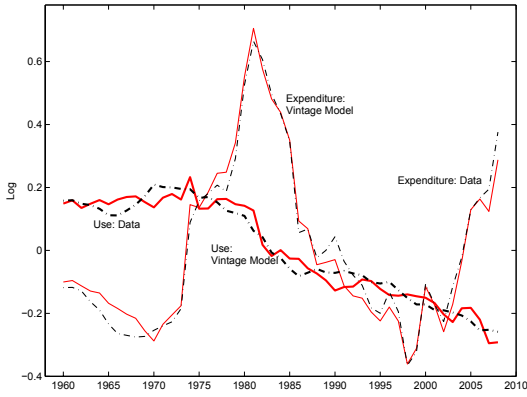


(a) Energy use, E/VA , energy expenditure, pE/VA , and the price of energy, p . (b) Capital-energy ratio and capital-value added ratio.

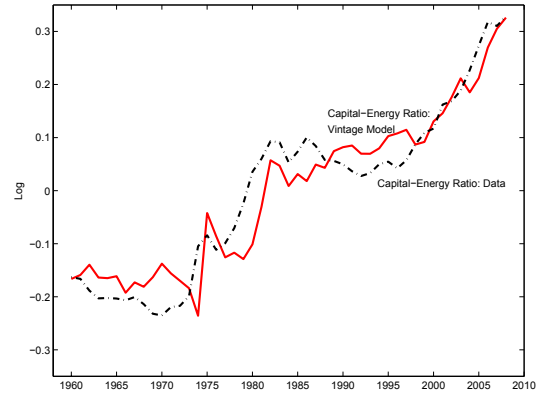


(c) Energy use, E , Value Added, VA , and the price of energy, p , for the US economy.

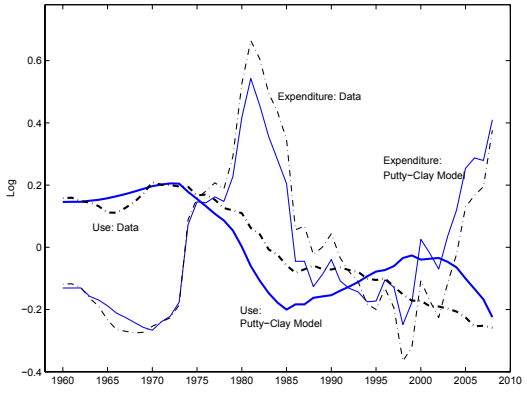
Figure 3: Energy use and energy efficiency in the US economy.



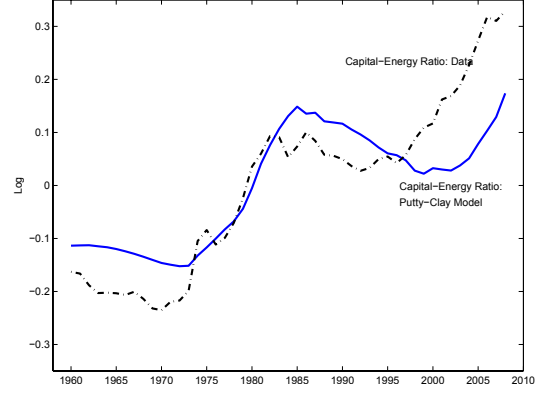
(a) Energy use and energy expenditure



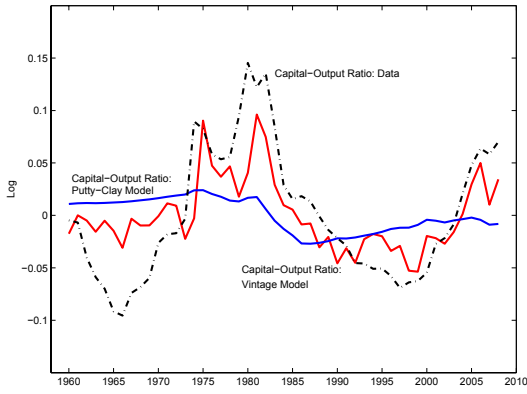
(b) Capital to energy ratio



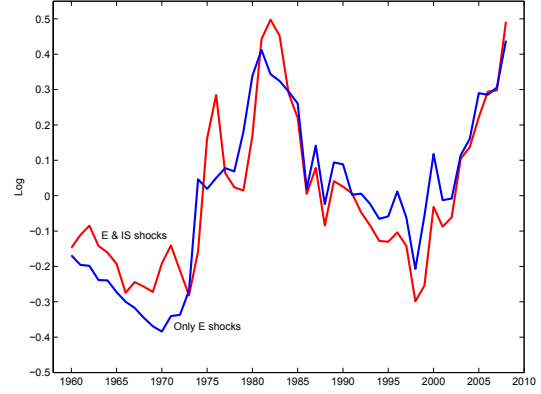
(c) Energy use and energy expenditure



(d) Capital to energy ratio

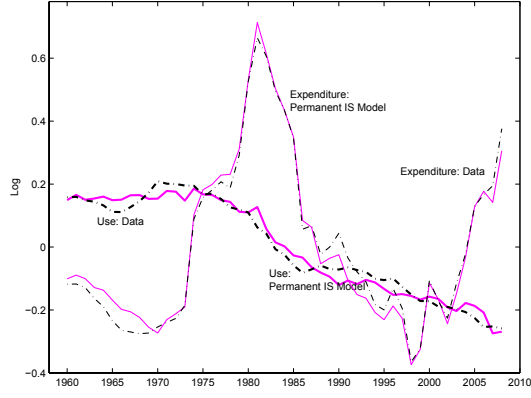


(e) Capital to value added ratio

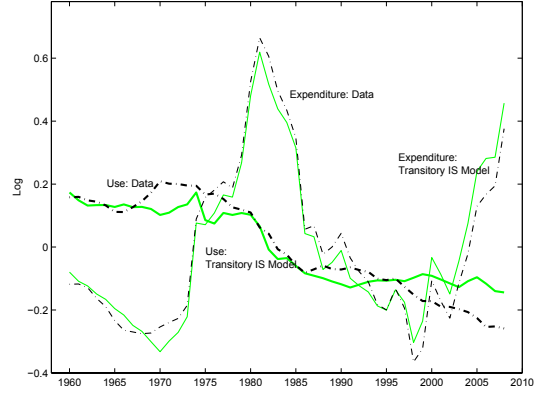


(f) Efficiency of the new capital goods, v_{t+1}^{-1}

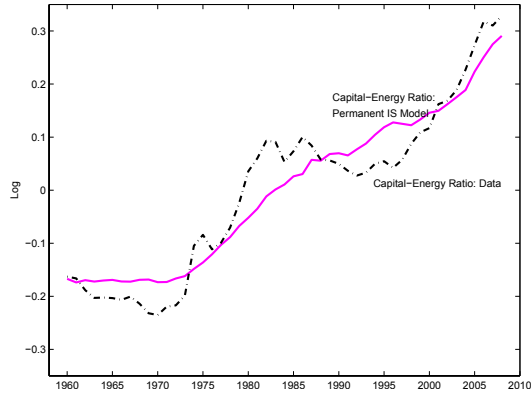
Figure 4: The benchmark economy (red line), the economy without ISTC shocks (blue line), and the data (black line).



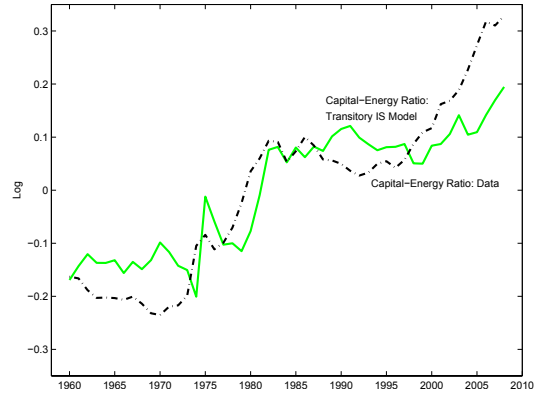
(a) Energy use and energy expenditure



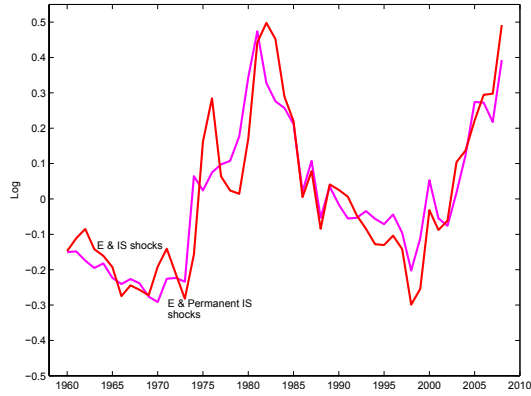
(b) Energy use and energy expenditure



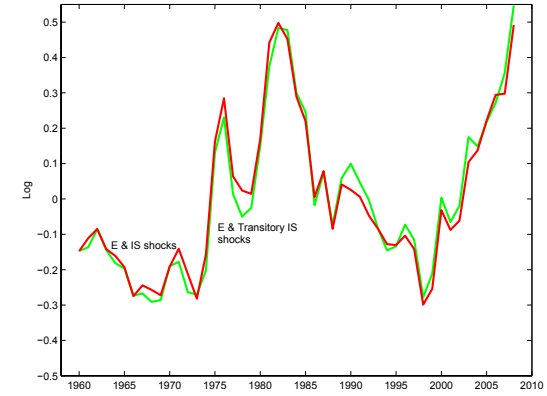
(c) Capital to energy ratio



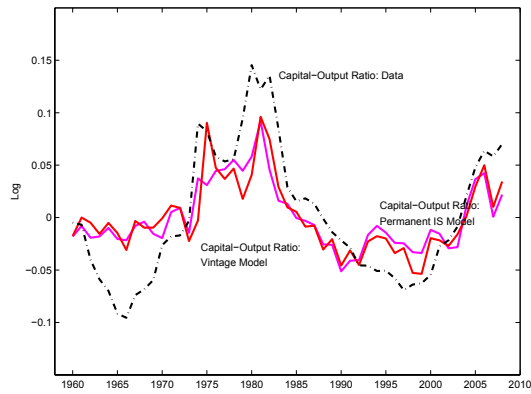
(d) Capital to energy ratio



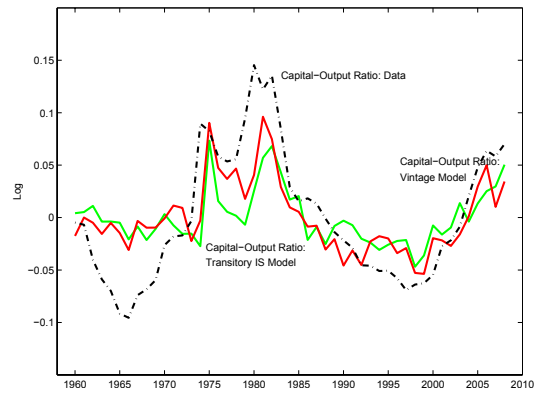
(e) Efficiency of the new capital goods, v_{t+1}^{-1}



(f) Efficiency of the new capital goods, v_{t+1}^{-1}



(g) Capital to value added ratio



(h) Capital to value added ratio

Table 1: Aggregate targets

Param.	Observation	Value
Preferences		
β	$K/(VA + pE) = 2.66$	0.9545
Technology		
α	$wL/(VA + pE) = 55.94\%$	0.4406
ϖ	$I/(VA + pE) = 28.41\%$	0.0523
\tilde{v}^μ	$pE/(VA + pE) = 4.51\%$	0.0160
μ	energy share in vintage $t = \alpha/\mu$	16.2409

Notes: Average for period 1960-2008. $u(c) = \ln c$, VA = measured GDP + services of consumer durables + services of public capital – VA of energy producing sectors.

Table 2: State space representation of embodied and disembodied technical progress. Full sample: 1947-2009, and restricted sample: 1959-2009, in annual and quarterly frequency.

	λ	θ	ρ_λ	ρ_θ	σ_λ	σ_θ
Quarterly Data	0.0073	0.0023	0.6622	0.2856	0.0063	0.0052
47:04 - 09:01	(0.0010)	(0.0001)	(0.0002)	(0.0001)	(0.0001)	(0.0001)
Quarterly Data	0.0078	0.0028	0.7223	0.2633	0.0054	0.0049
59:04 - 09:01	(0.0006)	(0.0001)	(0.0004)	(0.0002)	(0.0001)	(0.0001)
Annual Data	0.0303	0.0092	0.0368	0.2336	0.0282	0.0139
1947 - 2009	(0.0079)	(0.0005)	(0.0137)	(0.0020)	(0.0026)	(0.0004)
Annual Data	0.0323	0.0112	0.1444	0.1463	0.0272	0.0125
1959 - 2009	(0.0119)	(0.0007)	(0.0114)	(0.0021)	(0.0053)	(0.0006)
Observations	(1) 246	(2) 198	(3)	(4) 61	(5) 49	(6)
LL	(1) -1831	(2) -1522	(3)	(4) -306	(5) -253	(6)
Prob>F	0.000	0.000		0.000	0.000	

Notes: Standard errors in parentheses. All estimates are significant at 1%. F-test for joint significance.

References

- Acemoglu, D. (2002). Directed technical change. *Review of Economic Studies* 9(4), 781–809.
- Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012, February). The environment and directed technical change. *American Economic Review* 102(1), 131–66.
- Atkeson, A. and P. J. Kehoe (1999). Models of energy use: Putty-putty versus putty-clay. *American Economic Review* 89(4), 1028–43.
- Baumeister, C. and G. Peersman (2012). The role of time-varying price elasticities in accounting for volatility changes in the crude oil market. *Journal of Applied Econometrics* 27(4).
- Berndt, E. R. and D. O. Wood (1975). Technology, prices and the derived demand for energy. *Review of Economics and Statistics* 57(3), 259–68.
- Blanchard, O. J. and J. Galí (2007, January). The macroeconomic effects of oil price shocks: Why are the 2000s so different from the 1970s? In *International Dimensions of Monetary Policy*, NBER Chapters, pp. 373–421. National Bureau of Economic Research, Inc.
- Bureau of Economic Analysis (2003). Fixed assets and consumer durable goods in the United States, 1925–97. <http://www.bea.gov>.
- Cooley, T. F. and E. C. Prescott (1995). Economic growth and business cycles. In T. F. Cooley (Ed.), *Frontiers of Business Cycle Research*, Chapter 1, pp. 1–38. Princeton: Princeton University Press.
- Cummins, J. G. and G. L. Violante (2002). Investment-specific technical change in the us (1947–2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics* 5(2), 243–284.
- Díaz, A., L. A. Puch, and M. D. Guilló (2004). Costly capital reallocation and energy use. *Review of Economic Dynamics* 7(2), 494–518.
- Edelstein, P. and L. Kilian (2007). The response of business fixed investment to changes in energy prices: A test of some hypotheses about the transmission of energy price shocks. *The B.E. Journal of Macroeconomics* 7(1), 35.
- Fisher, J. D. M. (2006, June). The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy* 114(3), 413–451.
- Gordon, R. J. (1990). *The Measurement of Durable Goods Prices*. Chicago: University of Chicago Press: National Bureau of Economic Research Monograph Series.
- Gordon, R. J. (1996). *NBER Macroeconomics Annual*, Volume 11, Chapter Can Technology Improvements Cause Productivity Slowdowns?: Comment, pp. 259–267. The University of Chicago Press.
- Gort, M., J. Greenwood, and P. Rupert (1999, January). Measuring the rate of technological progress in structures. *Review of Economic Dynamics* 2(1), 207–230.
- Greenwood, J., Z. Hercowitz, and P. Krusell (1997). Long-run implications of investment-specific technological change. *American Economic Review* 87(3), 342–62.
- Greenwood, J., Z. Hercowitz, and P. Krusell (2000, January). The role of investment-specific technological change in the business cycle. *European Economic Review* 44(1), 91–115.
- Griffin, J. M. and P. R. Gregory (1976). An intercountry translog model of energy substitution responses. *American Economic Review* 66(5), 845–57.

- Hamilton, J. D. (2008). *The New Palgrave Dictionary of Economics*, Chapter Oil and the Macroeconomy. Houndsmills, U.K. and New York: Palgrave Macmillan.
- Hassler, J., P. Krusell, and C. Olovsson (2012, October). Energy-saving technical change. Working Paper 18456, National Bureau of Economic Research.
- Justiniano, A., G. Primiceri, and A. Tambalotti (2011). Investment shocks and the relative price of investment. *Review of Economic Dynamics* 14(1), 101–121.
- Kilian, L. (2008, December). The economic effects of energy price shocks. *Journal of Economic Literature* 46(4), 871–909.
- Knittel, C. R. (2011). Automobiles on steroids: Product attribute trade-offs and technological progress in the automobile sector. *American Economic Review* 101(7), 3368–99.
- Metcalf, G. E. (2008). An empirical analysis of energy intensity and its determinants at the state level. *The Energy Journal* 29(3), 1–26.
- Newell, R. G., A. B. Jaffe, and R. N. Stavins (1999). The induced innovation hypothesis and energy-saving technological change. *The Quarterly Journal of Economics* 114(3), 941–975.
- Pindyck, R. S. (1979). Interfuel substitution and the industrial demand for energy: An international comparison. *Review of Economics and Statistics* 29(2), 173–89.
- Pindyck, R. S. and J. J. Rotemberg (1983). Dynamic factor demands and the effects of energy price shocks. *American Economic Review* 73(5), 1066–79.
- Popp, D. (2002). Induced innovation and energy prices. *American Economic Review* 92(1), 160–180.
- Rodríguez-López, J. and J. L. Torres (2012, February). Technological sources of productivity growth in germany, japan, and the united states. *Macroeconomic Dynamics* 16(01), 133–150.
- Sato, K. (1976). The meaning and measurement of the real value added index. *The Review of Economics and Statistics* 58(4), 434–442.
- Solow, R. M. (1960). *Mathematical Methods in Social Sciences, 1959: Proceedings of the First Stanford Symposium*, Chapter Investment and technological progress, pp. 89–104. Stanford University Press.
- Steinbuks, J. and K. Neuhoﬀ (2010). Operational and investment response to energy prices in the OECD manufacturing sector. Cambridge Working Papers in Economics 1015, Faculty of Economics, University of Cambridge.